ALTERNATING CURRENT PROBLEMS

RICHARD E. BROWN

ALTERNATING CURRENT PROBLEMS

Digitized by the Internet Archive
in 2025

https://archive.org/details/bwb_S0-EKW-885

ALTERNATING CURRENT PROBLEMS

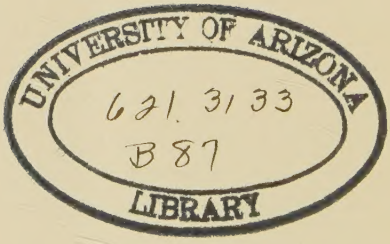
BY

RICHARD E. BROWN

Assistant Professor of Electrical Engineering
Moore School of Electrical Engineering
University of Pennsylvania

NEW YORK
JOHN WILEY & SONS, INC.
LONDON: CHAPMAN & HALL, LIMITED
1925

Copyright, 1925
BY
RICHARD E. BROWN



PRESS OF
BRAUNWORTH & CO.
BOOK MANUFACTURERS
BROOKLYN, N. Y.

PREFACE

THE object of this book is to illustrate by problems the fundamental principles of Electrical Engineering encountered by students in courses on alternating current machinery. It has been the aim of the author to illustrate the fundamental principles many times by relatively few problems. The time required to solve the problems is not more than that available to the average student. In problems on machinery, frequent reference is made to the A.I.E.E. Standardization Rules. This book is intended to be used in connection with a text-book or a lecture course.

R. E. BROWN.

September 24, 1924.

62179

TABLE OF CONTENTS

Harmonic Voltages and Currents in Series and Parallel Circuits. Problems 1-19, 34-38, and 46-47.

Symbolic Notation. Problems 20-33, and 39-45.

Equivalent Series and Parallel Circuits. Problems 34-36, and 46.

Power Measurements. Problems 48-62, and 121.

Power-Factor Correction. Problems 47, 179, 186, and 230.

Calculation of the Constants of Series and Parallel Circuits from Test Data. Problems 63 and 64.

Equivalent Y-Phase and Delta-Phase Circuits. Problems 65-67.

Non-Harmonic Voltages and Currents in Series and Parallel Circuits. Problems 69-81.

Wave Analysis. Problems 76-79, and 88.

Voltage, Current, and Flux Relations in Transformers. Problems 82-103, 115, and 116.

Constants of a Transformer by Test. Problems 106, and 109.

Equivalent Circuits of Transformers. Problems 104-112.

Voltage Regulation and Efficiency of Transformers. Problems 112-114, 117-120, 122, and 130.

Parallel Operation of Transformers. Problems 123, and 124.

Auto Transformers. Problems 125-129.

Transformer Connections. Problems 131-136.

Flux, Voltage, Current, and Frequency Relations of Synchronous Generators. Problems 137-155.

Armature Reaction in Synchronous Generators. Problems 145, and 146.

Voltage Regulation and Efficiency of Synchronous Generators. Problems 153-161, and 171.

Parallel Operation of Synchronous Generators. Problems 162-170.

Synchronous Motors Performance. Problems 172-188.

Synchronous Converters Performance. Problems 189-205.

Polyphase Induction Motors Performance. Problems 206-231, and 236.

Polyphase Induction Motors Connected in Concatenation. Problems 232-235.

Induction Generators Performance. Problems 237-239.

Single-Phase Induction Motor. Problems 240-243.

Single-Phase Series Motor. Problem 244.

Compensated Repulsion-Induction Motor. Problem 245.

Fynn Synchronous-Induction Motor. Problem 246.

Unbalanced Three-Phase Circuits. Problems 68, and 247.

Phase-Balancer. Problem 247.

ALTERNATING CURRENT PROBLEMS

1. Define: (*a*) effective resistance; (*b*) inductive reactance; (*c*) condensive reactance; (*d*) impedance.

2. A sinusoidal electromotive force of the form $e = 150 \sin (377t + 6)$ exists in a closed circuit of which the resistance is 4 ohms. The circuit has no reactance. (*a*) Write an equation for the instantaneous current in the circuit. (*b*) Determine: (1) the phase difference between the electromotive force and the current; (2) the frequency of the e.m.f.; and (3) the frequency of the current. (The angle $377t + 6$ is expressed in radians when t is expressed in seconds.)

3. A 60-cycle e.m.f. of the form $e = 100 \sin (2\pi ft)$ exists in a coil whose terminals are connected together. The electric circuit of the coil has a self-inductance of 0.1 henry and has no appreciable resistance or capacity. The power loss in the magnetic circuit of the coil is negligible. (*a*) Write an equation for the instantaneous current in the coil. (*b*) Determine: (1) the phase difference between the e.m.f. and the current; and (2) the power-factor of the coil.

4. A voltage drop of the form

$$v = \sqrt{2}\, 440 \sin (2\pi ft + 3)$$

exists across the terminals of a perfect 25-microfarad condenser. (*a*) Write an equation for the instantaneous current through the condenser. (*b*) Determine: (1) the phase difference between the voltage and the current; and (2) the power-factor of the condenser.

5. Prove that the average power in a single-phase circuit is equal to the product of the r.m.s. value of the line voltage by the r.m.s. value of the line current by the power-factor of the circuit. What assumptions did you make?

6. A 60-cycle current of the form

$$i = 120 \sin (2\pi \text{ ft.} + \phi)$$

is established in a coil having a resistance of 1.732 ohms and a total inductive reactance (the reactance due to all of the flux linking the coil) of 1 ohm. The power loss in the magnetic circuit of this coil is negligible. (*a*) Write the equation for the total voltage drop across the terminals of the coil. (*b*) Determine the power-factor of the coil. (*c*) Construct a vector diagram showing the current, the e.m.f. due to the coil flux, the voltage drop due to the coil resistance, and the total voltage drop across the terminals of the coil.

Answers.—(*a*) $v = 240 \sin \left(2\pi ft + \phi + \frac{\pi}{6}\right)$. (*b*) Power-factor 0.866. (*c*) See the diagram Fig. 1.

In Fig. 1 the voltage drop and the e.m.f. due to the flux are considered as positive in the same direction in which the current is assumed as positive.

7. A 60-cycle voltage of the form

$$v = 100 \sin\left(2\pi ft - \frac{\pi}{4}\right)$$

exists across the terminals of a condenser of 42 microfarads capacity. The power loss in the condenser is 25 watts. (*a*) Write the equation for the instantaneous current through the condenser. (*b*) Calculate the power-factor of the condenser. (*c*) Construct a vector diagram showing the voltage drop across the condenser and the current. Assume the voltage drop positive in the same direction in which the current is assumed as positive.

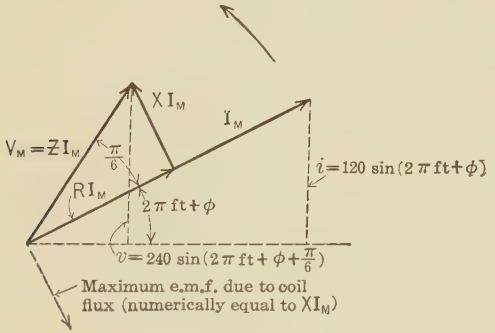

XI_M
I_M
$V_M = ZI_M$
$\frac{\pi}{6}$
RI_M
$2\pi ft + \phi$
$i = 120 \sin(2\pi ft + \phi)$
$v = 240 \sin(2\pi ft + \phi + \frac{\pi}{6})$
Maximum e.m.f. due to coil flux (numerically equal to XI_M)

Fig. 1.

8. An air-core coil is connected to a 110-volt (r.m.s. value), 60-cycle supply. The line current is 2.45 amperes (r.m.s. value) and the power loss in the coil is 81.2 watts. (*a*) Calculate the total inductance of the coil. (*b*) Calculate the power loss in the coil at 110 volts, 25 cycles. (*c*) What assumptions did you make in part (*b*)? (*d*) Construct a complete vector diagram.

9. A 100-microfarad condenser is connected to a 110-volt, 60-cycle supply. The power loss in the dielectric of the condenser is 40 watts. Calculate: (*a*) the charging current; (*b*) the leakage current; and (*c*) the power-factor of the condenser.

10. A reactor having a total inductance of 205 millihenries and an effective resistance of 1.5 ohms is connected to a 250-volt (constant r.m.s. voltage) supply of variable frequency. (*a*) Plot a curve showing the relation between current and frequency. (*b*) What assumptions did you make in (*a*)? Under what conditions are your assumptions correct?

11. Define: (*a*) conductance; (*b*) susceptance; (*c*) admittance.

12. Construct a voltage drop-current vector diagram for a portion of a circuit consisting of resistance and inductance in series. Show on your diagram the vectors ZI, rI, xI, YV, gV, and bV. (*a*) With the aid of this diagram, prove the following relations:

(1) Impedance, $Z = \sqrt{r^2 + x^2}$.

(2) Resistance, $r = \dfrac{g}{g^2 + b^2}$.

(3) Reactance, $x = \dfrac{b}{g^2 + b^2}$.

(4) Admittance, $Y = \sqrt{g^2 + b^2}$.

(5) Conductance, $g = \dfrac{r}{r^2 + x^2}$.

(6) Susceptance, $b = \dfrac{x}{r^2 + x^2}$.

(*b*) Repeat (*a*) for a portion of a circuit consisting of a perfect condenser in series with a resistance.

(*c*) Repeat (*a*) for a portion of a circuit consisting of a leaky condenser.

13. A 5-microfarad condenser having no losses is connected to a 2200-volt (constant r.m.s. voltage) supply of variable frequency. (*a*) Plot a curve showing the relation between line current and frequency. (*b*) Plot a similar curve for the condenser when it is shunted by a non-inductive resistance of 2000 ohms.

14. An air-core reactor having a constant effective resistance of 1 ohm and a constant self-inductance of 0.8 henry is connected in series with a perfect condenser of 52-microfarads capacity to a 100-volt (r.m.s. value) constant voltage supply. (*a*) Plot a curve showing the relation between r.m.s. values of current and frequency. (*b*) What is the maximum r.m.s. current? (*c*) What is the frequency and the power-factor when the current is a maximum? (*d*) Draw a complete voltage drop-current vector diagram showing the voltage and current relations when the current is a maximum.

15. The reactor and condenser specified in Problem 14 are connected in parallel to a 100-volt constant-voltage a.c. supply. (*a*) Plot a curve showing the relation between r.m.s. values of line current and frequency. (*b*) At what frequency and power-factor is the r.m.s. line current a minimum? (*c*) Draw a complete voltage drop-current vector diagram showing the relation between the voltage and the currents when the line current is a minimum.

16. Repeat the solutions of Problems 14 and 15.

The condenser is now assumed shunted by a resistance of 5000 ohms.

17. A portion of a circuit consists of two parts, A and B, connected in series. The voltage across A is 75 volts, and that across B is 105 volts, and that across A and B combined 82 volts. (*a*) Draw a vector diagram showing the voltage relations. (*b*) What kind of a combination will give these voltage relations?

18. A parallel circuit has two branches, A and B. The current in A is 25 amperes, that in B 33 amperes, and that in A and B combined 33 amperes. (*a*) Draw a vector diagram showing the relations between the currents. (*b*) What kind of a combination will give these current relations?

19. A coil having a resistance of 5 ohms and a self-inductance of 0.7 henry is connected in series with a 10-microfarad condenser having negligible loss. The circuit is connected to a source of power of which the frequency may be varied. (*a*) Plot curves showing the relation between inductive-reactance and frequency, condensive-reactance and frequency, and total reactance and frequency. (*b*) Plot curves showing the relation between inductive-susceptance and frequency, condensive-susceptance and frequency, and total susceptance and frequency.

20. A vector of 100 units length leads a reference axis by 60 degrees. Write the symbolic expression for the vector referred to the reference axis.

21. A vector of 100 units length lags a reference axis by 60 degrees. Write the symbolic expression for the vector referred to the reference axis.

22. The following measurements were made on a branch of a circuit:

Current through the branch, 42 amperes.

Voltage drop across branch, 215 volts.

Power-factor of branch, 0.82 leading.

Determine: (*a*) the symbolic expression for the current referred to the voltage; and (*b*) the symbolic expression for the voltage referred to the current.

23. The voltage drop, referred to the current, across a portion of a series circuit is $102 - j21$ volts. (*a*) What is the numerical value of the current? (*b*) Does the current lead or lag the voltage? (*c*) What is the power-factor of this portion of the circuit?

24. An alternating current generator supplies a load having two parts, A and B, connected in series. The symbolic expressions for the voltages referred to the current, are; for part A $(11.2 - j7)$, and for part B $(27 + j6)$. (*a*) What is the symbolic expression for the total voltage across A and B? (*b*) What is the numerical value of this voltage? (*c*) Determine: (1) the power-factor of part A; (2) the power-factor of part B; and (3) the power-factor of A and B combined.

25. A certain alternator supplies a load consisting of two parts, A and B, connected in parallel. The symbolic expressions for the currents, referred to the voltage drop across the parallel branches, are for part A $(11.3 + j55)$, and for part B $(1 - j8.2)$. (*a*) What is the symbolic expression for the line current referred to the load voltage? (*b*) Determine: (1) the power-factor of part A; (2) the power-factor of part B; and (3) the power-factor of A and B combined.

26. Show that the symbolic expression for the impedance of any part of a circuit is $r + jx$, provided to j is assigned the value $\sqrt{-1}$. (r represents the resistance and x the reactance.)

27. Show that the symbolic expression for the admittance of any part of a circuit is $g - jb$, provided to j is assigned the value $\sqrt{-1}$. (g represents the conductance and b the susceptance.)

28. Prove that the symbolic expression for the impedance of a part of a circuit consisting of resistance and inductance is $r + j(+x)$.

29. Prove that the symbolic expression for the impedance of a series circuit consisting of a pure resistance and perfect condenser is $r + j(-x)$.

30. Prove that the symbolic expression for the admittance of two parallel branches, consisting of a pure resistance in parallel with a pure inductance, is $g - j(+b)$, where g is the conductance and b the susceptance.

31. Prove that the symbolic expression for the admittance of a leaky condenser is $g - j(-b)$.

32. An impedance of $2 + j5$ ohms is connected in series with an impedance of $9 + j3$ ohms to a 120-volt a.c. generator having an assumed impedance of zero. (*a*) What is the impedance of the series circuit? (*b*) What is the symbolic expression for the current in the circuit?

33. An impedance of $4 + j1$ ohms is connected in parallel with an impedance of $1 - j3$ ohms to a 55-volt a.c. generator of negligible impedance. (*a*) Calculate the admittance of the circuit. (*b*) What is the symbolic expression for the line current? (*c*) Determine the power in this circuit. (Use the symbolic method in calculating the power.)

34. An air-core reactor having an effective resistance of 0.5 ohm and a self-inductance of 280 millihenries is connected to a 60-cycle supply. (*a*) What is the symbolic expression for the impedance of the reactor?

(*b*) Calculate the constants of a parallel circuit equivalent in every way to the circuit of the reactor at 60 cycles. Assume that the parallel circuit consists of a pure resistance in parallel with a pure inductance. (*c*) How would a change in frequency alter the constants calculated in (*b*)?

35. An impedance of $4 + j1$ ohms is connected in parallel with an impedance of $1 - j3$ ohms to a 60-cycle a.c. supply. (*a*) Calculate the admittance of the circuit. (*b*) Calculate the constants of a series circuit equivalent to the parallel circuit at 60 cycles. Assume that the equivalent series circuit contains a pure resistance and a pure inductance or perfect condenser. (*c*) Plot a curve showing the relation between the reactance of the equivalent series circuit and the frequency.

36. The following results were obtained by a test on a leaky condenser:

Applied voltage, 220 volts;

Frequency, 60 cycles;

Capacity, 75 microfarads;

Power loss in the condenser, 52 watts.

(*a*) Calculate the constants of a series circuit equivalent to the leaky condenser at 60 cycles. (*b*) Calculate the constants of a parallel circuit equivalent to the leaky condenser at 60 cycles.

37. A pure resistance and a coil having negligible resistance are connected in parallel across a 220-volt, 60-cycle supply. The current through the resistance is 14 amperes and that through the coil 10 amperes. The total line current is 16 amperes. Calculate the frequency which would result in equal currents in the coil and the resistance.

38. A single-phase transmission line delivers 50 kilowatts at 2200 volts. The load power-factor is 0.75. The line copper loss is 8 per cent of the power delivered. (*a*) What is the value of the line resistance? (*b*) Calculate the copper loss when the same power is transmitted over the same line to a unity power-factor load.

39. Prove that multiplying the symbolic expression for a vector by the operator $(\cos\theta + j\sin\theta)$ results in the symbolic expression for a vector θ degrees ahead of the original vector and equal in length to the original vector.

40. Prove that multiplying the symbolic expression for a vector by the operator $(\cos\theta - j\sin\theta)$ results in the symbolic expression for a vector θ degrees lagging the original vector and equal in length to the original vector.

41. Prove that $86.6 + j50 = 100\epsilon^{j\frac{\pi}{6}}$.

42. Prove that the symbolic expression for the impedance of an electric circuit is equal to the product of the numerical value of the impedance and $\epsilon^{j\theta}$, where $\tan\theta = \frac{x}{r}$.

43. Prove that the symbolic expression for the admittance of an electric circuit is equal to the product of the numerical value of the admittance and $\epsilon^{j(-\theta)}$, where $\tan\theta = \frac{b}{g}$.

44. The symbolic expression for the voltage across an electric circuit is $100\epsilon^{j\beta}$. The impedance of the circuit is $20\epsilon^{j\theta}$. Show that the current in the circuit is equal to $5\epsilon^{j(\beta-\theta)}$.

45. The symbolic expression for the voltage across an electric circuit is $130\epsilon^{j\beta}$. The admittance of the circuit is $2\epsilon^{j(-\theta)}$. Show that the symbolic expression for the current is

$$260\epsilon^{j(\beta-\theta)} = 260\,[\cos(\beta-\theta) + j\sin(\beta-\theta)].$$

46. A given series circuit has three parts, A, B, and C. Part A is a non-inductive, non-condensive resistance of 5 ohms, part B has an effective resistance of 2 ohms and an inductance of 0.15 henry, and part C is a perfect condenser of 25 microfarads capacity. The voltage across the whole circuit is 440 volts and the frequency is 60 cycles. (*a*) What is the power-factor of this circuit? (*b*) Determine the constants of a series circuit which in every way is equivalent to the given series circuit at 60 cycles. (*c*) Determine the constants of a parallel circuit equivalent in every way to the series circuit at 60 cycles. (*d*) Construct a vector diagram showing the line voltage; the line current; the components of the voltage in phase, and 90 degrees displaced from the line current; and the components of the current in phase, and 90 degrees displaced from the line voltage.

47. A reactor having an effective resistance of 0.5 ohm and an inductive reactance of 5 ohms is connected in series with an incandescent lamp load having an effective resistance of 10.3 ohms. This load is connected to a single-phase transmission line having a total effective resistance of 0.52 ohm and a total inductive reactance of 0.7 ohm. The terminal voltage of the generator supplying this load is 150 volts. The frequency is 60 cycles. Determine the capacity of a perfect condenser which when connected to the ter-

minals of the alternator will make the power-factor of the circuit unity (power-factor measured at the generator terminals).

48. Show by diagram the best arrangement of meters for measuring the power and power-factor of a load in which the power is small, the current small, and the voltage relatively high. This question is based on the assumption that no correction is made for the power consumed by the meters themselves.

49. Show by diagram the best arrangement of meters for measuring the power and the power-factor of a circuit in which the power is small, the current large, and the voltage relatively small. This question is based on the assumption that no correction is to be made for the power consumed by the meters themselves.

50. A voltmeter is connected to measure the voltage across a certain 60-cycle circuit. Stray fields are produced in the vicinity of the voltmeter by, (*a*) direct current, (*b*) 60-cycle alternating current, (*c*) 58-cycle alternating current, and (*d*) 30-cycle alternating current. What influence have these fields on the indication of the meter? How are instrument errors due to stray fields avoided?

51. Prove that the power in an *N-wire* polyphase transmission system may be measured by N-1 wattmeters.

52. Prove that the average power in a balanced three-phase system is equal to $\sqrt{3}\ VI \cos\theta$, where V is the line voltage, I the line current, and $\cos\theta$ the Y-phase or delta-phase power-factor.

53. Prove that the power in a balanced or unbalanced three-phase system may be measured by two wattmeters.

54. Draw a diagram showing the proper way to connect two wattmeters for measuring the power in a three-phase system. Draw a vector diagram showing the line voltages, the line currents, the currents in the current coils of the meters, and the voltages across the potential coils of the meters. Assume that the three-phase load is balanced. Mark the vectors on your diagram in such a way that it is possible to tell in what directions you have assumed the voltages and currents as positive.

55. Two wattmeters are connected for measuring the power in a balanced three-phase system. How would you determine experimentally whether the meter indications should be added or subtracted?

56. Two wattmeters are connected to measure the power in a balanced three-phase system. Prove that one of the meter indications is $VI \cos (30 + \theta)$, and the other $VI \cos (30 - \theta)$. Show that the indication of one of the meters is zero when the power-factor is 0.5.

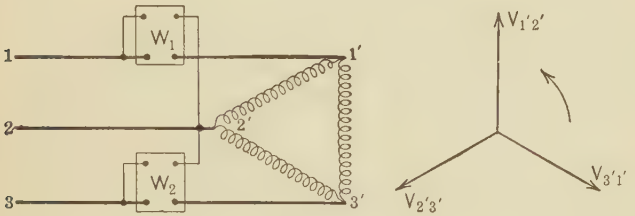


Fig. 2.

57. Figure 2 represents two similar wattmeters, connected symmetrically, for measuring the power in a balanced delta-connected load. The phase rotation of the voltages is as indicated by the vector diagram at

the right. The r.m.s. line voltage is 220 volts. The delta-phase power-factor is 0.82 lagging, and the total transmitted load is 50 kilowatts. (*a*) Calculate the power indications of W_1 and W_2. (*b*) Has the phase rotation of the line voltages anything to do with the indications of the meters?

58. A wattmeter and a Y-box are connected for measuring the power in a balanced three-phase circuit. See Fig. 3. The resistance of the potential circuit of the meter is Rp, and the resistance of each leg of the Y-box is r. (*a*) Derive a formula, in terms of Rp, r, v_{12},

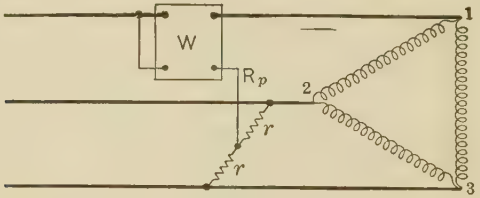


FIG. 3.

and v_{13}, for the instantaneous current in the potential coil of the meter. (*b*) Derive a formula for the factor by which the indication of the wattmeter must be multiplied to obtain the total power in the circuit.

59. In Fig. 3, $Rp = 1000$ ohms, $r = 850$ ohms, and the power indicated by the meter is 2000 watts. Calculate the total power delivered to the load.

60. An electrodynamometer wattmeter is connected as shown in Fig. 4. The resistance of the potential circuit of the meter is 1662 ohms, and its self-inductance is 0.0082 henry. The meter as connected indicates accurately on d.c. loads. What will be the power

indication of the meter when used for measuring 75 watts true power in a circuit operating at, (*a*) 60 cycles unity power-factor, (*b*) 60 cycles 0.10 leading power-factor, (*c*) 60 cycles 0.10 lagging power-factor?

61. Two similar wattmeters are connected for measuring the reactive volt-amperes of a balanced three-phase load. The current coil of meter A is connected in line 1, and its potential coil is connected across lines 2 and 3. The current coil of meter B is connected in line 3, and its potential coil is connected across lines 2 and 1. (*a*) Draw a diagram showing the proper con-

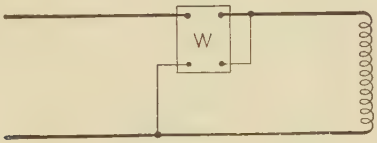


FIG. 4.

nections. (*b*) Prove that the indications of the meters are proportional to the reactive volt-amperes.

62. Two similar wattmeters are connected symmetrically for measuring the power in a balanced three-phase load. One of the meters indicates 7580 watts and the other 2458 watts. (*a*) Calculate the power-factor of the load. (*b*) Calculate the phase-angle between, (1) the delta-phase voltage and the delta-phase current, (2) the Y-phase voltage and the Y-phase current, and (3) the voltage between lines 1 and 2 and the current in line 1. (*c*) How many answers are possible in part (3) of (*b*)?

63. The following measurements were made on a single-phase circuit of which the constants are unknown:

Voltage	Frequency	Current	Power
100	d.c.	0	0
100	50 cycles	1.75 amps.	6.1 watts
100	69 cycles	50.0 amps. Maximum Current	5000.0 watts
100	80 cycles	3.82 amps.	29.2 watts

(*a*) Determine the nature of the circuit and the constants of the same. (*b*) What assumptions did you make?

64. The following measurements were made on a circuit of which the constants are unknown:

Voltage	Frequency	Current	Power
100	10 cycles	13.42 amps.	720 watts
100	60 cycles	2.64 amps.	27.95 watts
100	d.c.	25.0 amps.	2500.00 watts

(*a*) Calculate the constants of the circuit. (*b*) What assumptions did you make in (*a*)?

65. A balanced three-phase, Y-connected load requires 500 kilowatts of power. The line voltage is 440 volts and the power-factor of the load is 0.85 lagging. (*a*) Calculate the resistance and the reactance of each leg of the Y-connected load. (*b*) Calculate the resistance and the reactance of each delta-phase of a delta-connected load equivalent in every way to the given Y-connected load.

66. The following measurements were made on a balanced three-phase load:

Line Voltage	Frequency	Current	Power
102 volts, 3-phase 40 volts between any two terminals	60 cycles d.c.	10 amps. 11.5 amps.	400 watts

(*a*) Calculate the d.c. resistance per Y-phase and per delta-phase. (*b*) Calculate the equivalent effective resistance and the equivalent reactance per Y-phase and per delta-phase. (*c*) Why is the resistance to a.c. greater than the d.c. resistance?

67. A balanced Y-connected load is connected in parallel with a balanced delta-connected load to a 220-volt, 60-cycle, balanced three-phase supply. The impedance of each Y-phase is $1.2 + j0.8$. The impedance of each delta-phase is $3.2 + j5.1$. (*a*) Calculate the power and the power-factor of the combined load. (*b*) Calculate the constants of a single Y-connected load equivalent in every way to the two loads on a 60-cycle circuit.

68. A Y-connected generator is connected to an unbalanced Y-connected load as shown in Fig. 5. The Y-phase terminal voltages of the generator each have an r.m.s. value of 52 volts and are 120 degrees out of phase. Each line has a resistance of 0.5 ohm and an inductive reactance of 0.42 ohm. The impedances of the legs of the Y-phase load are: $Z_{0'1'} = 4 + j0$, $Z_{0'2'} = 1 + j0$, and $Z_{0'3'} = 1.4 + j7$. Calculate the currents in the three lines when the phase ro-

tation of the generator voltages is, (*a*) $V_{01} - V_{02} - V_{03}$, and (*b*) $V_{01} - V_{03} - V_{02}$.

69. The voltage applied across the terminals of a perfect 61-microfarad condenser contains a fundamental and a third harmonic. The frequency of the fundamental is 50 cycles and its maximum value is 200 volts. The maximum value of the third harmonic is 50 volts. (*a*) Calculate the maximum values of the harmonics in the current wave. (*b*) What is the r.m.s. value of the resultant charging current?

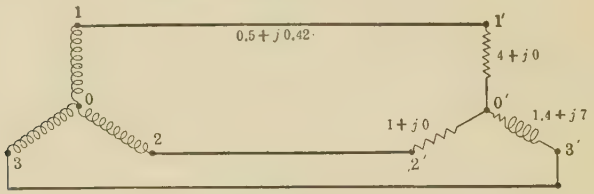


FIG. 5

70. A non-sinusoidal voltage is applied across the terminals of a coil having an effective resistance of 0.52 ohm and an inductance of 0.11 henry. The fundamental of the voltage has a frequency of 60 cycles and a maximum value of 100 volts. The third harmonic in the voltage has a maximum value of 30 volts. (*a*) Calculate the maximum values of the fundamental and third harmonic currents. (*b*) What is the r.m.s. value of the total current supplied to the coil?

71. A non-sinusoidal voltage of the form

$$v = 440 \sin (2\pi 60t) + 220 \sin 3\left(2\pi 60t + \frac{\pi}{6}\right)$$

is applied across the terminals of a perfect 20-microfarad condenser. (*a*) Determine the equation for the

current through the condenser. (*b*) Determine the r.m.s. value of the total current supplied to the condenser.

72. A non-sinusoidal voltage of the form

$$v = 125 \sin\left(2\pi 50t + \frac{\pi}{8}\right) + 52 \sin 3\left(2\pi 50t + \frac{\pi}{6}\right)$$

is applied to a coil having an effective resistance of 0.52 ohm and a self-inductance of 0.11 henry. (*a*) Determine the equation for the current through the coil. (*b*) Calculate the power absorbed by the coil.

73. A sinusoidal voltage of the form

$$v = 321 \sin\left(2\pi 60t + \frac{\pi}{3}\right)$$

produces a current of the form

$$i = 245 \sin\left(2\pi 60t + \frac{\pi}{4}\right) + 32 \sin 3\left(2\pi 60t + \frac{\pi}{9}\right)$$

Calculate the average power supplied to the circuit. Explain your method of calculating the power.

74. A 60-cycle and a 25-cycle alternator are connected in series with each other to a load having a resistance of 1 ohm and a total self-inductance of 0.005 henry. The terminal voltage of each alternator is 120 volts r.m.s. value. Calculate: (*a*) the r.m.s. value of the line current; (*b*) the r.m.s. value of the line voltage; (*c*) the power output of each alternator.

75. A direct-current generator and a 50-cycle alternator are connected in series with each other to a load having an effective resistance of 4.5 ohms. The terminal voltage of each generator is 120 volts (r.m.s. value). Calculate: (*a*) the r.m.s. value of the line

current; (*b*) the average value of the line current; (*c*) the power transmitted to the load; (*d*) the r.m.s. value of the line voltage.

76. In a symmetrical rectangular wave the base of the half-wave is 3 inches and the width is 1 inch. (*a*) Determine the maximum value of the fundamental. (*b*) Determine the maximum value of the third harmonic.

77. The voltage and current waves of a certain resistance load contain a fundamental and a third harmonic. Prove that the power absorbed by the load is equal to $r(I_1^2 + I_3^2)$, where I_1 and I_3 are the r.m.s. values of the fundamental and third harmonic currents.

78. Why are no even harmonics present in a symmetrical wave? Why do alternators generally produce symmetrical voltage waves?

79. Thirty-six equally spaced ordinates 10 degrees apart, obtained from a full wave length of a non-symmetrical current wave, have the following values:

i_0	0	i_{120}	22	i_{240}	− 7
i_{10}	6	i_{130}	19.5	i_{250}	− 8
i_{20}	11	i_{140}	17.3	i_{260}	− 8.8
i_{30}	16	i_{150}	15.5	i_{270}	−10.0
i_{40}	20	i_{160}	13.5	i_{280}	−11.8
i_{50}	23.2	i_{170}	11.0	i_{290}	−12.4
i_{60}	26	i_{180}	8	i_{300}	−12.4
i_{70}	27.5	i_{190}	5	i_{310}	−12
i_{80}	28	i_{200}	1	i_{320}	−10
i_{90}	28	i_{210}	−2	i_{330}	− 7.8
i_{100}	27	i_{220}	−5	i_{340}	− 5.0
i_{110}	25	i_{230}	−6	i_{350}	− 2.3
				$i_{360} = i_0$	=0

The frequency of the fundamental of the above wave is 60 cycles. Determine the equation of the current wave.

80. Under what conditions will a sinusoidal voltage produce a non-sinusoidal current wave?

81. Why are the higher harmonics in voltage and current waves usually undesirable?

82. The flux linking each turn of a coil on a transformer varies sinusoidally and has a maximum value of 10^6 lines. The coil has 100 turns and the frequency of the flux is 60 cycles per second. Calculate the r.m.s. value of the induced e.m.f.

83. The terminal voltage of a certain coil on a transformer varies sinusoidally. Prove that the flux linking the coil also varies sinusoidally. (The rI drop in a coil of a transformer is usually very small compared with the terminal voltage.)

84. The maximum flux linking a transformer coil having N turns is ϕ_m. The flux varies sinusoidally at a frequency of f-cycles per second. Derive formulas for calculating: (*a*) the average e.m.f. induced in the coil; and (*b*) the effective e.m.f. induced in the coil.

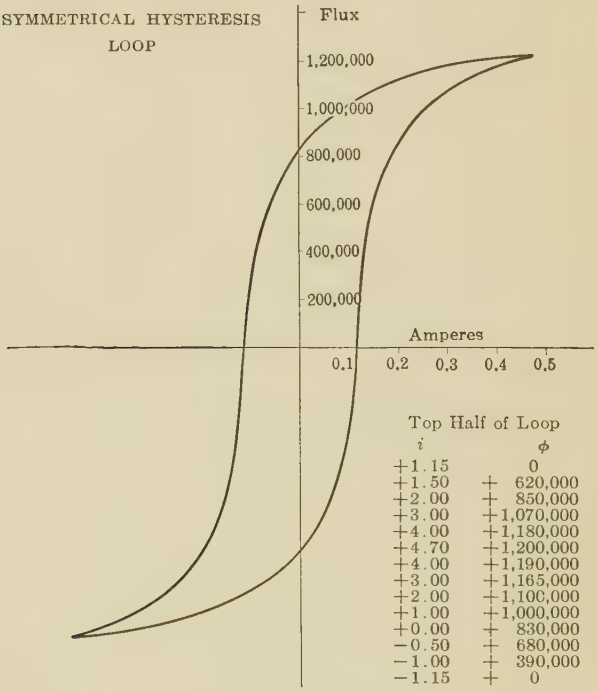
85. The high-voltage winding of a transformer has 1000 turns and the low-voltage winding has 250 turns. What is the turn ratio? (See A.I.E.E. Standardization Rules in regard to the turn ratio.)

86. The voltage applied to the primary winding of a transformer is 440 volts and the secondary full-load terminal voltage at unity power-factor load is 2200 volts. (*a*) Calculate the voltage ratio (see A.I.E.E. Standardization Rules). (*b*) Why is the ratio 2200/440 different than the turn ratio?

87. Under what conditions of load is the ratio of the

current in the low-voltage winding of a transformer to the current in the high-voltage winding approximately equal to the turn ratio?

88. Figure 6 is a curve showing the relation between



Top Half of Loop

i	ϕ
+1.15	0
+1.50	+ 620,000
+2.00	+ 850,000
+3.00	+1,070,000
+4.00	+1,180,000
+4.70	+1,200,000
+4.00	+1,190,000
+3.00	+1,165,000
+2.00	+1,100,000
+1.00	+1,000,000
+0.00	+ 830,000
−0.50	+ 680,000
−1.00	+ 390,000
−1.15	+ 0

Fig. 6.

the flux and the (magnetizing + hysteresis) component of the exciting current of a transformer.

(*a*) Draw the wave of the (magnetizing + hysteresis) component of the exciting current required to produce a sinusoidal flux of 1,200,000 lines maximum value.

(*b*) Analyze the wave obtained in (*a*) for the fundamental and the higher harmonics. (*c*) How serious is the error made in assuming the exciting current of a transformer a sinusoidal quantity?

89. A coil wound on an iron core consumes 1150 watts when connected to a source of a.c. power. The voltage is 124 volts and the current is 40.8 amperes, r.m.s. values. The resistance of the coil is 0.15 ohm. (*a*) How much power is transferred from the coil to the magnetic circuit? (*b*) What is the phase angle between the current and the flux? (*c*) Construct a vector diagram showing the relations between the current, the flux, the applied voltage, the induced e.m.f., and the rI drop. Use the double subscript method for indicating the positive sense on all vectors. (Assume that the current, the flux, and the voltage are sinusoidal quantities, and assume that there is no leakage flux.)

90. In a certain coil 100 turns are linked by a maximum flux of 1,000,000 lines, 200 turns by 800,000 lines, and 150 turns by 500,000 lines. The flux is produced by a 60-cycle current. Calculate the r.m.s. value of the total e.m.f. induced in the coil. (Assume that the flux and current are sinusoidal quantities and that the e.m.f.'s induced in all turns are in phase.)

91. At 110 volts and 60 cycles the total iron loss in a certain reactor is 324 watts. The hysteresis loss is 210 watts. (*a*) Calculate the total iron loss when the voltage and the frequency are both increased 20 per cent. (*b*) Calculate the total iron loss when the frequency and the voltage are both lowered 20 per cent.

92. The total iron loss in a reactor is 846 watts. The hysteresis loss is 550 watts. Calculate the total

iron loss in a reactor having laminations of one-half the thickness but identical in every other way to the reactor having 846 watts loss.

93. The following measurements were made on a transformer at no-load:

Primary Voltage	Primary Frequency	Primary Current	Secondary Voltage
110 r.m.s.	60	0.52	440 r.m.s.

(*a*) Calculate the value of the mutual inductance. (*b*) What assumptions did you make in (*a*)? Why?

94. (*a*) Prove that when there is a transfer of electrical power from an electric circuit to a magnetic circuit or to another electric circuit the flux linking the first circuit and the current in it cannot be in phase. (*b*) Why is the leakage flux in a transformer winding nearly in phase with the current through the winding?

95. The power loss in the iron core of a certain reactor is 1000 watts. The applied voltage to the reactor is 220 volts. The line current is 11.2 amperes. Calculate: (*a*) the r.m.s. value of the counter e.m.f.; and (*b*) the phase angle between the current and the flux.

96. In a transformer what factors determine, (*a*) the exciting current, (*b*) the total flux linking the primary winding, (*c*) the total flux linking the secondary winding, and (*d*) the efficiency?

97. In a constant-potential transformer, why is the primary power-factor practically equal to the secondary power-factor?

98. Derive approximate formulas for the voltage and current ratios of a constant-potential transformer. Under what conditions of load are your formulas approximately correct?

99. The following things are known about a 2 kv-a. transformer:

Turn ratio, 4;

Exciting current, 0.52 ampere;

Core loss, 21 watts;

D.C. resistance of primary winding, 0.06 ohm;

Turns on primary winding, 60;

Maximum mutual flux linking primary and secondary windings, 690,000 lines (assumed to vary sinusoidally at a frequency of 60 cycles per sec.);

Maximum leakage flux of primary, 17,400 (assumed to vary sinusoidally);

Power loss due to leakage flux (assumed negligible).

(*a*) Calculate the r.m.s. value of the induced e.m.f. in the primary winding due to the change in linkages produced by the mutual flux.

(*b*) Calculate the r.m.s. value of the induced e.m.f. in the primary due to the leakage flux.

(*c*) Determine the r.m.s. value of the primary terminal voltage.

(*d*) Construct a vector diagram showing the exciting current, the mutual flux, the leakage flux, the induced e.m.f. in the primary due to the mutual flux, the induced e.m.f. in the primary due to the leakage flux, the resistance drop due to the exciting current, and the total applied primary voltage.

100. In a given step-down transformer the following quantities are known:

Mutual inductance, M;

Primary resistance r_1 and leakage reactance x_1;

Secondary resistance r_2 and leakage reactance x_2;

Turn ratio a;

Exciting current I_e;

(*a*) Prove that the primary applied voltage

$$\dot{V}_1 = (r_1 + jx_1)\,\dot{I}_1 + a\,\dot{E}_2$$

Where $\dot{I}_1$ = total primary current and $\dot{E}_2$ = the secondary induced e.m.f. due to mutual flux.

(*b*) Prove that the secondary terminal voltage

$$\dot{V}_2 = \dot{E}_2 - (r_2 + jx_2)\,\dot{I}_2$$

Where I_2 = the secondary current.

(*c*) Prove that the secondary induced voltage

$$\dot{E}_2 = jwM\dot{I}_m$$

Where I_m is the magnetizing component of the exciting current.

101. Figures 7 and 8 show two possible ways of constructing a transformer. The two transformers have an equal number of turns on their primary windings; the turns on the secondary windings are also equal in number. One of the transformers has a solid iron core and the other a laminated iron core. The primary voltages and frequencies are the same in each transformer.

Compare the two transformers as to:

(*a*) Total magnetic flux linking the primary;

(*b*) Primary currents and secondary voltages at no-load;

(*c*) Primary currents and secondary voltages when the secondary windings carry equal load currents;

(*d*) Iron losses. (Assume that the iron cores have the same cross-section and weight.)

(*e*) Copper losses. (Assume that the same size wire is used on both transformers and that the mean length of the turns are the same.)

(*f*) Efficiencies.

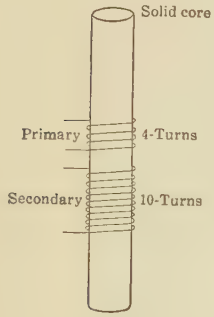


Fig. 7.

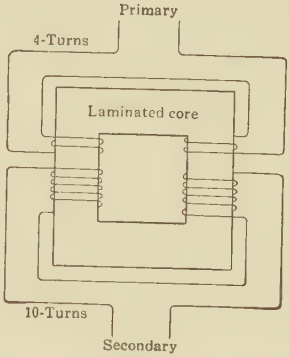


Fig. 8.

102. A 550-kv-a., 11,000 to 2200-volt, 60-cycle transformer has the following constants:

Effective resistance of high-voltage winding 0.75 ohm;

Leakage reactance of high-voltage winding, 3.2 ohms;

Effective resistance of low-voltage winding, 0.0321 ohm;

Leakage reactance of low-voltage winding, 0.153 ohm;

The above constants were measured at 75 degrees C.;

Turn ratio, 5.

The transformer supplies 550 kv-a. to a load operating at 2200 volts and 0.90 lagging power-factor. The working temperature of the transformer is 75 degrees C. Calculate: (*a*) the secondary current; (*b*) the secondary induced e.m.f.; (*c*) the primary induced voltage; (*d*) the primary current (the exciting current in the primary is 0.32 ampere and lags behind the primary voltage drop due to the e.m.f. induced by the mutual flux by an angle whose cosine is 0.321); (*e*) the total voltage which must be applied to the primary in order to obtain 2200 volts at the load (this voltage probably is not 11,000 volts); and (*f*) the efficiency.

103. The primary winding of a 75 kv-a., 2500- to 320-volt, 25-cycle, step-down transformer has an effective primary resistance of 0.492 ohm and a primary leakage reactance of 0.68 ohm. The losses in the core due to the mutual flux are 1520 watts. At 2500 volts the exciting current is 2.1 amperes. The number of turns on the primary winding is 219. The secondary winding is open-circuited. Calculate: (*a*) the induced e.m.f. in the primary winding due to the mutual flux; (*b*) the phase angle between this induced e.m.f. and the exciting current; (*c*) the value of the mutual flux and the phase angle between the mutual flux and the exciting current. (Assume that the flux varies sinusoidally.)

104. A step-down transformer has a turn ratio of 4. The primary voltage is 442 volts and is held constant. The load on the secondary requires 20 amperes at 80 volts. The required 80 volts at the load is obtained by inserting a 1.5-ohm rheostat in the secondary leads. Show that the proper voltage can also be obtained at

the load by inserting a 24-ohm rheostat in the primary leads and no rheostat in the secondary leads.

105. A certain transformer has a turn ratio of 3. Prove that the insertion of an inductive reactance of 9 ohms in the high-voltage leads is equivalent to the insertion of an inductive reactance of 1 ohm in the low-voltage leads.

106. (*a*) Give a method for measuring the core-loss conductance and the magnetizing susceptance of a transformer. (See A.I.E.E. Rules in regard to the proper method for measuring the no-load losses in a transformer.)

(*b*) Give a wiring diagram showing the best way of grouping the instruments for making the no-load test on a transformer. (Assume that no corrections are to be made for the power consumed by the instruments themselves.)

107. In a 13,200- to 440-volt transformer the core-loss conductance and the magnetizing susceptance were measured on the low-voltage side. The turn ratio of the transformer is 30. (*a*) Show how to calculate the core-loss conductance and the magnetizing susceptance referred to the high-voltage side. (*b*) Why is it desirable to measure the core-loss conductance and the magnetizing susceptance on the low-voltage side?

108. Given:

x_1 the leakage reactance of the high-voltage winding;

r_1 the effective resistance of the high-voltage winding;

x_2 the leakage reactance of the low-voltage winding;

r_2 the effective resistance of the low-voltage winding;

g_2 the core-loss conductance measured on the low-voltage side;

b_2 the magnetizing susceptance measured on the low-voltage side,

and

a the turn ratio.

Derive the exactly equivalent circuit, the constants being referred to (*a*) high-voltage side, and (*b*) low-voltage side.

109. Outline a method for measuring the effective resistance and the leakage reactance of a transformer. (See A.I.E.E. Rules in regard to the proper method of conducting the load-loss test on a transformer.) Show how the instruments should be grouped in this test. (Assume that no corrections are to be made for the power consumed by the instruments themselves.)

110. Show how the exactly equivalent circuit of a transformer must be modified to obtain the approximate circuit. What errors are made in assuming the approximate circuit?

111. The following tests were made on a 2 kv-a. 110- to 440-volt, 60-cycle transformer:

No-Load Test. (Performed on Low-Voltage Side)

Applied Voltage	Current	Power	Frequency
110	0.52	20	60

Load-Loss Test. (Meters Connected in the High-Voltage Side)

Applied Voltage	Current	Power	Frequency
17.6	4.55	45	60

Turn ratio, 4;

D.C. resistance high-voltage winding, 0.84 ohm;

D.C. resistance low-voltage winding, 0.072 ohm;

Temperature at which the above measurements were made, 18° C.

Determine: (*a*) the ratio of the effective resistance to the d.c. resistance of the windings; (*b*) the constants of a circuit exactly equivalent to the transformer at 18° C. (The constants are to be referred to high-voltage side.) (*c*) the constants of a circuit exactly equivalent to the transformer at 18° C. (The constants are to be referred to low-voltage side); and (*d*) the constants of a circuit approximately equivalent to the transformer at 18° C. (The constants are to be referred to the high-voltage side.)

112. The following are the constants of a 20 kv-a. 2200- to 220-volt, 60-cycle step-down transformer (see the A.I.E.E. Rules in regard to the rating of transformers):

Turn ratio, 10;

Effective resistance of the high-voltage winding at 25° C. = 1.47 ohm;

Effective resistance of the low-voltage winding at 25° C. = 0.023 ohm;

Ratio effective to d.c. resistance, 1.15;

Core-loss, 164 watts (measured on the high-voltage side at an induced voltage of 2200 volts in the high-voltage winding);

Exciting current, 0.48 amp. (measured on the high-voltage side at an induced voltage of 2200 volts in the high-voltage winding);

Total reactance drop in the transformer, 2.84 per cent (see A.I.E.E. Rules);

Assume the primary leakage reactance referred to the secondary the same as the secondary leakage reactance.

The transformer delivers rated kv-a. to an 85 per cent lagging power-factor load. The temperature of the transformer is 75° C. Calculate: (*a*) the voltage which must be applied to the high-voltage side in order to obtain rated voltage at the load (use the exactly equivalent circuit in your calculations); (*b*) the efficiency of the transformer for the conditions of load specified in (*a*).

113. Solve Problem 112, using the approximately equivalent circuit. Refer all constants to low-voltage side. Compare the results with those in Problem 112.

114. The following things are known about a 5000-kv-a. 140,000- to 5000-volt, 60-cycle, single-phase transformer:

D.C. resistance of high-voltage winding, 11.2 ohms at 32° C.;

D.C. resistance of low-voltage winding, 0.019 ohm at 32° C.;

Turn ratio, 28;

Per cent reactance drop, 8.63;

Iron loss, 41,500 watts;

Load-test loss (copper loss), 38,900 watts, measured at 32° C.;

Per cent exciting current 7.00.

(*a*) Determine the constants of a circuit approximately equivalent to this transformer at 75° C. Refer all constants to low-voltage turns.

(*b*) The transformer is used as a step-down transformer and delivers rated kv-a. load to a circuit oper-

ating at 90 per cent leading power-factor. Determine the primary voltage necessary.

115. Construct a complete vector diagram for a single-phase transformer.

Solution: (See Fig. 9.)

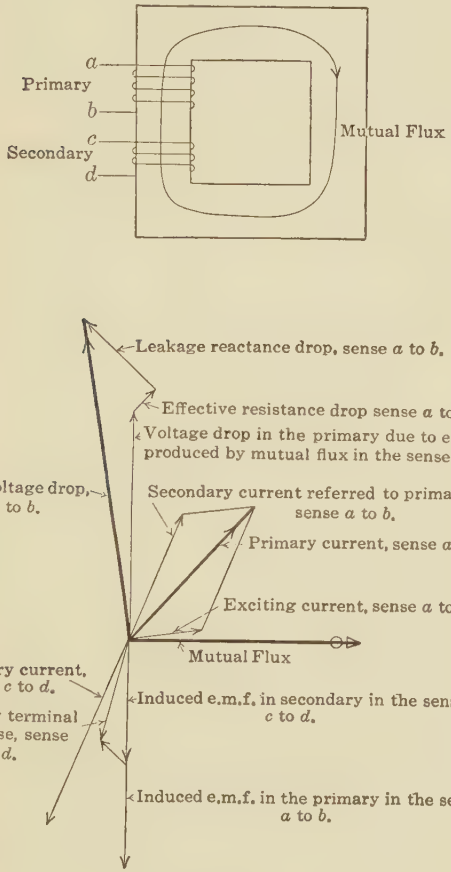

a
Primary
b
c
Secondary
d
Mutual Flux
Leakage reactance drop, sense a to b.
Effective resistance drop sense a to b.
Voltage drop in the primary due to e.m.f. produced by mutual flux in the sense a to b.
Primary voltage drop, sense a to b.
Secondary current referred to primary turns, sense a to b.
Primary current, sense a to b.
Exciting current, sense a to b.
Mutual Flux
Secondary current, sense c to d.
Secondary terminal voltage rise, sense c to d.
Induced e.m.f. in secondary in the sense c to d.
Induced e.m.f. in the primary in the sense a to b.

FIG. 9.

116. Figure 10 represents a lagging device used on a.c. watt-hour meters. A correct indication is obtained on the meter when the flux Φ and the voltage V are 90 time degrees out of phase. This phase relation is obtained by properly adjusting the resistance R. Construct a complete vector diagram showing the voltage, current, and flux relations of the two coils when the resistance R has the proper value.

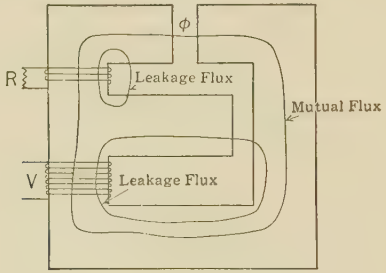


Fig. 10.

117. The following things are known about a 2000-kv-a., 140,000- to 5000-volt, 60-cycle transformer:

D.C. resistance of high-voltage winding, 36.7 ohms at 75° C.;

D.C. resistance of low-voltage winding, 0.060 ohm at 75° C.;

Turn ratio, 28.

Load-loss (copper loss at 75° C.), 19,100 watts;

Per cent leakage reactance drop, 8.98;

Per cent exciting current, 6.98;

Iron-loss, 24,000 watts.

The transformer is used as a step-down transformer.

(*a*) Calculate the regulation of the transformer at rated kv-a. load and at a temperature of 75° C. for,

(1) unity power-factor load, (2) 80 per cent leading power-factor load, (3) 80 per cent lagging power-factor load. Use the methods outlined in the A.I.E.E. Standardization Rules.

(*b*) Draw a complete vector diagram showing the voltage and current relations for each load specified in (*a*).

118. The following things are known about an 833-kv-a., 13,200- to 6600-volt, 60-cycle transformer:

D.C. resistance of high-voltage winding, 1.066 ohms at 75° C.;

D.C. resistance of low-voltage winding, 0.276 ohm at 75° C.;

Per cent reactance drop, 4.54;

Turn ratio, 20;

Load-loss (copper loss at 75° C.), 9490 watts;

Exciting current, 5.61 per cent;

Iron-loss, 6350 watts.

The transformer is used as a step-up transformer.

(*a*) Calculate the voltage regulation of this transformer at rated kv-a. load and at a temperature of 75° C. for, (1) unity power-factor load, (2) 80 per cent leading power-factor load, and (3) 80 per cent lagging power-factor load. Use the methods outlined in the A.I.E.E. rules.

(*b*) Construct the complete vector diagram showing the voltage and current relations for each load specified in (*a*).

(*c*) Calculate the efficiency of the transformer for each load specified in (*a*).

119. The following things are known about a 4-kv-a., 2200- to 1000-volt, 60-cycle constant-current transformer:

Effective resistance of high-voltage winding, 16.7 ohms;

Effective resistance of low-voltage winding, 5.5 ohms.

No-Load Test

Primary Voltage	Primary Current	Core-loss	Frequency
2200 volts	0.3 amp.	149 watts	60

Turn ratio, 1.80

Impedance Test

	Movable Coil Position	Impedance, ohms
1	Primary and secondary coils together	270
2		385
3		500
4		615
5		730
6		845
7	Primary and secondary coils apart	960

The values given above are all referred to 75° C.

The secondary current (the current in the low-voltage winding) is maintained constant at 4 amperes. The power-factor of the load is unity. The primary voltage is maintained constant at 2200 volts; the frequency is also constant. Plot curves showing the relation between: (*a*) secondary watts output and coil position; (*b*) primary current and coil position; (*c*) primary power-factor and coil position; and (*d*) efficiency and coil position.

120. A 100-ampere to 5-ampere ratio current transformer is used for measuring the current in a 60-cycle power circuit. The ammeter is connected to the 5-ampere winding of the transformer. The indication

of the ammeter is 4.88 amperes. The impedance of the ammeter is 0.17 ohms. The mutual inductance between the primary and the secondary windings of the transformer is 0.0178 henry. (*a*) Calculate the secondary terminal voltage when the ammeter indication is 4.88 amperes. (*b*) Calculate the secondary terminal voltage when the ammeter is disconnected (secondary winding open-circuited) and the primary current is the same as in (*a*). (*c*) Compare the relative magnitudes of the exciting currents in parts (*a*) and (*b*). (*d*) What assumptions did you make in (*b*)?

121. An ammeter, voltmeter, and wattmeter are connected for measuring the power and the power-factor of a single-phase load. The voltage and the current are stepped-down by instrument transformers. The following things are known:

Wattmeter indication, 64 watts (the wattmeter is accurate on circuits where no instrument transformers are used);
Ammeter indication, 2.3 amperes (accurate);
Voltmeter indication, 112.3 volts (accurate);
Ratio and phase-angle of voltage transformer obtained from calibration curves; ratio—9.94, phase-angle—0.16° (secondary voltage lags primary voltage);
Ratio and phase-angle of current transformer obtained from calibration curves; ratio—8.12, phase-angle—1.3 degrees (the secondary current leads the primary current).

1. Calculate: (*a*) the line voltage and the line current; (*b*) the apparent power and power-factor of the circuit; (*c*) the true power and power-factor of the

circuit; and (*d*) the per cent error in the power indication due to the error introduced by the phase-angles of the transformers.

2. The line voltage is maintained constant and the load so adjusted that the product of the voltmeter and ammeter indications equals the wattmeter indication, equals 64 watts. What will be the per cent error in the wattmeter indication due to the errors introduced by the instrument transformers? (Assume that the ratios and phase-angles of the transformers are the same as in 1. This assumption is probably never strictly true.)

122. The following things are known about a 5000-kv-a., 140,000- to 5000-volt, 60-cycle, step-down transformer:

Rated current of the primary winding, 37 amperes;

Exciting current at rated voltage, 2.59 amperes.

No-Load Test

Applied Voltage, Per Cent	Exciting Current, Per Cent	Iron Loss, Per Cent	Frequency, Per Cent
100	100	100	100
150	650	195	100

Iron loss at rated voltage, 41,500 watts;

Load-test loss (copper loss at 32° C.) = 38,900 watts;

D.C. resistance of low-voltage winding, 0.019 ohm at 32° C.;

D.C. resistance of high-voltage winding 11.2 ohms at 32° C.;

Turn ratio, 28;

Per cent reactance drop, 8.63;

The transformer is loaded in such a way that the secondary induced e.m.f. is in phase with the secondary current.

1. Calculate: (*a*) the efficiency of the transformer when operating at rated primary voltage, current, and frequency; (*b*) the efficiency of the transformer when operating at 150 per cent rated voltage, and at rated primary current and frequency.

2. Calculate the primary power-factor for each case stated in (1).

123. Two 140,000- to 5000-volt, 60-cycle transformers are operated in parallel on both the primary and the secondary sides. The following things are known about the transformers:

Rating	2000 kv-a.	5000 kv-a.
Effective resistance high-voltage winding	20.7 ohms	12.4 ohms at operating temp.
Effective resistance low-voltage winding	0.08 ohm	0.024 ohm at operating temp.
Iron loss	24,000 watts	41,500 watts
Exciting current	7.1 per cent	6.9 per cent
Reactance drop	8.85 per cent	8.4 per cent
Turn ratio	28	28

The transformers are operated at a secondary voltage of 5000 volts at all loads.

Calculate: (*a*) the circulating current in the secondary windings at no load; (*b*) the primary voltage, current, and power-factor when the combined load on the transformers is 4000 kv-a. at 97 per cent lagging power-

factor; (*c*) the primary voltage, current, and power-factor when the combined load on the transformers is 7000 kv-a. at 97 per cent lagging power-factor; and (*d*) the value of the impedance which must be inserted in the secondary leads of one of the transformers to make them share the load properly in (*c*).

124. Two 60-cycle, 6600- to 66,000-volt, step-up transformers are operated in parallel on the primary and secondary sides. The following data on the transformers were obtained from the load-loss test. (The instruments were connected in the high-voltage side.)

Transformer Capacity	Current	Voltage	Power Loss at 75° C.
1000 kv-a.	15.15 amps.	3350 volts	8,120 watts
5000 kv-a.	75.8 amps.	3700 volts	29,000 watts

Turn ratio, 10.

Calculate: (*a*) the kv-a. output of each transformer when the total load is 6000 kv-a. at 90 per cent leading power-factor; (*b*) the maximum 90 per cent leading power-factor kv-a. load possible without exceeding the rated capacity of either transformer; and (*c*) the value of the impedance which must be connected in series with the secondary winding of each transformer to obtain the proper division of load at all loads.

125. Figure 11 is a wiring diagram of an auto-transformer. The rating of an auto-transformer is usually expressed as the difference between the voltage of the high-voltage winding and the voltage of the low-voltage winding multiplied by the rated current of the portion

of the winding across which this difference in voltage exists. The volt-ampere rating of the transformer in Fig. 11 is equal to $(V_h - V_l)\, I_{12}$.

Derive the following approximate formula for the relative ratings of an ordinary two-circuit transformer and an auto-transformer for equal power transformations:

$$\frac{\text{Rating of two-circuit transformer}}{\text{Rating of auto-transformer}} = \frac{V_h}{(V_h - V_l)}.$$

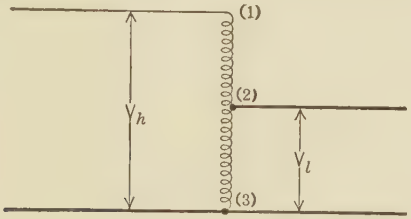


Fig. 11.

126. A certain single-phase load requires 75 kv-a. at 440 volts. The only power supply available is a 600-volt a.c. circuit. Calculate: (*a*) the power rating of a two-circuit transformer required to supply this load; and (*b*) the power rating of an auto-transformer required to supply the same load.

127. (*a*) Repeat the calculations in Problem 126 for a load of 75 kv-a. at 100 volts.

(*b*) What danger is involved in using an auto-transformer?

128. The following things are known about a 20-kv-a., 500 to 100-volt, 60-cycle auto-transformer:

Effective resistance at 75° C. of high-voltage winding (1–3) Fig. 11 = 0.071 ohm.

Effective resistance at 75° C. of low-voltage winding (2–3) = 0.0054 ohm.

Leakage reactance of winding (1–2) = 0.261 ohm.

Leakage reactance of winding (2–3) = 0.0241 ohm

Iron loss, 250 watts.

No-load primary current, 1.75 amperes.

The load is connected across terminals 2 and 3 and requires 15 kv-a. at a lagging power-factor of 0.84. The temperature of the transformer is 75° C.

Calculate: (*a*) the voltage regulation of the transformer; and (*b*) the efficiency of the transformer.

129. Give a wiring diagram showing the proper way to connect a two-step, three-phase, auto-starter to a three-phase motor. The motor is to be started at one-half rated voltage.

130. The daily load on an 830-kv-a., 13,200- to 6600-volt, 60-cycle transformer is as follows:

500 kv-a., at 6600 volts, 84 per cent lagging power factor on for 4 hours.

410 kv-a., at 6600 volts, 75 per cent lagging power factor on for 7 hours.

150 kv-a., at 6600 volts, 75 per cent lagging power factor on for 8 hours.

No-load at 6600 volts, on for 5 hours.

The copper loss, obtained from the load-loss test, is 9200 watts at 65°;

The iron loss is 6300 watts at all loads;

The exciting current is 7 per cent.

Calculate the approximate all-day efficiency of this transformer. Assume that the temperature of the transformer does not differ appreciably from 65° C. at any time during the day.

131. The primary windings of three identical trans-

formers are Y-connected and the terminals are connected to a balanced three-phase power supply. The secondary windings may be connected in delta or in Y. Each secondary winding has an effective resistance of 0.1 ohm and a leakage reactance of 0.21 ohm. The induced e.m.f. in each secondary winding due to the mutual flux contains a fundamental of 5000 volts and a third harmonic of 542 volts r.m.s. values. Calculate: (*a*) the secondary terminal voltage at no-load, secondaries delta-connected; (*b*) the circulating current in the mesh formed by the secondary windings when delta-connected; (*c*) the secondary terminal voltage at no-load, secondaries Y-connected; and (*d*) the Y-phase secondary voltages, secondaries Y-connected.

132. Two identical transformers are connected in open-delta on the primary and secondary sides. The transformers supply a 625 kv-a., balanced three-phase load. The voltage across any two line wires supplying the load is 550 volts. The power-factor of the load is 0.94 lagging. (*a*) Calculate the kv-a. rating of the transformers required to supply this load. (*b*) Calculate the power-factor of each transformer secondary. (*c*) Draw a complete vector diagram showing all voltage and current relations.

133. Two identical transformers are used for the purpose of transforming from two-phase to three-phase. Each secondary winding is provided with 86.6 and 50 per cent taps. The secondaries are T-connected and supply a 550-kv-a., balanced three-phase, 60 per cent lagging power-factor load. The load voltage at the terminals of the transformers is 275 volts. The ratio of the turns on the high-voltage winding to the total number of turns on the low-voltage winding is 10.

(*a*) Calculate the approximate terminal voltages and currents in the primary windings. (Neglect the exciting currents and the impedances of the transformers.) (*b*) Draw a vector diagram showing the primary and secondary voltages and currents of each transformer. (*c*) Calculate the power and the power-factor of each transformer coil.

134. Figure 12 is a diagram of a mesh-connected, six-phase load. The voltages from *A*–*B*, *B*–*C*, *C*–*D*, *D*–*E*, *E*–*F*, and *F*–*A* have equal r.m.s. values and may

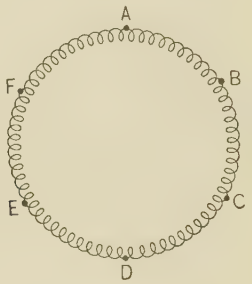


Fig. 12.

be represented by six vectors 60 time degrees apart. The r.m.s. value of these voltages is 150 volts. (*a*) Calculate the r.m.s. values of the voltages between *A* and *C*, *A* and *D*, and *A* and *E*. (*b*) Show by vector diagram that the instantaneous voltage between *A* and *C* in the sense *A*–*C* is equal to the instantaneous voltage between *F* and *D* in the sense *F*–*D*. In other words, show that the voltage *A*–*C* is in phase with the voltage *F*–*D*. (*c*) Show that voltage *C*–*E* is in phase with voltage *B*–*F* and equal to it. (*d*) Show that voltage *E*–*A* is in phase with voltage *D*–*B* and equal to

it. (*e*) Show that the voltages *A*–*D*, *C*–*F*, and *E*–*B* are equal and 120 degrees out of phase. (*f*) Show that the voltages *A*–*C*, *C*–*E*, and *E*–*A* are equal and 120 degrees out of phase. (*g*) Show that the voltages *F*–*D*, *D*–*B*, and *B*–*F* are equal and 120 degrees out of phase. Note that the six-phase voltages may be produced by combining two three-phase systems having opposite phase rotation of the line voltages.

135. Three identical 2200- to 440-volt single-phase transformers are connected in delta on the high-voltage side to a balanced three-phase supply. The secondaries supply a balanced six-phase load. The x_1 lead of transformer 1 (see A.I.E.E. Rules in regard to the proper way of marking the terminals of transformers) is connected to *A*, Fig. 12, and x_2 is connected to *D*. Similarly x_1 and x_2 of transformer 2 are connected to *C* and *F* respectively, and x_1 and x_2 of transformer 3 are connected to *E* and *B* respectively. The phase current (the current in *A*–*B* or *B*–*C*, etc.) has an r.m.s. value of 225 amperes. The phase power-factor is unity. (*a*) Calculate the phase voltage. (*b*) Determine the proper kv-a. rating of the transformers to supply this load.

136. Three identical 4400- to 600-volt single-phase transformers are Y-connected on the high-voltage side to a balanced three-phase supply. Each transformer has two 300-volt secondary windings. The secondaries are connected in double-delta to a balanced six-phase load. The mesh current in the six-phase load has an r.m.s. value of 310 amperes. The power-factor of the load is unity. Calculate: (*a*) the current ratings of the transformer windings; and (*b*) the total kv-a. rating of the transformers.

137. An alternator has 4 poles and is driven at a speed of 1800 r.p.m. Calculate the frequency.

138. Derive a formula for calculating the induced e.m.f. in a phase of an alternator. Assume that the field form is sinusoidal, that the coil pitch is 180 electrical degrees, and that the e.m.f.'s in all the coils are in phase.

139. Derive a formula for calculating the induced e.m.f. in a phase of an alternator. Assume that the field form is non-sinusoidal, that the coil pitch is not 180 electrical degrees and that the e.m.f.'s induced in the coils are not in phase.

140. Four coils are placed on the armature of an alternator as follows: Four of the coil sides are placed in adjacent slots, 15 electrical space degrees apart, and the remaining coil sides are similarly placed in adjacent slots located 180 electrical space degrees from the first. The coils are connected in series so as to obtain the maximum voltage between the terminals of the group. The e.m.f. induced in each coil is sinusoidal and has an r.m.s. value of 15.1 volts. (*a*) Calculate the resultant e.m.f. induced in the coil group. (*b*) What would be the resultant e.m.f. induced in the coil group if the coils were all placed in the same slots? (*c*) What are the advantages of a distributed armature winding over one that is concentrated?

141. A 4-pole, three-phase alternator has a double layer winding. The armature has four slots per pole per phase. There are four coil groups per phase and four coils per group (the coil sides are placed in adjacent slots). The coils in a group and the coil groups belonging to a phase are all connected in series. The coil pitch is $\frac{2}{3}\pi$. (*a*) Draw an armature winding dia-

gram. (*b*) Draw a section of the armature showing the location of the coil sides of a phase.

142. Prove that the third harmonic in the induced e.m.f. in an armature coil, due to the third harmonic in the space distribution of the flux, may be eliminated by making the coil pitch $\frac{2}{3}\pi$. What are the advantages and disadvantages of using short-pitch coils?

143. A two-phase alternator has 6 poles, operates at a speed of 1200 r.p.m., and has 600 turns in series per phase. The flux linking each turn varies sinusoidally and has a maximum value of 1,000,000 lines. The coil pitch is 180 electrical space degrees. The e.m.f.'s in the coils are in phase. Calculate: (*a*) the frequency of the induced e.m.f.; and (*b*) the r.m.s. value of the voltage induced between armature terminals.

144. Each phase of a four-pole, Y-connected alternator consists of four coil groups connected in series. Each coil group consists of four coils connected in series. The adjacent coil sides in a group are placed in adjacent slots 15 electrical space degrees apart. Each coil has 10 turns. The field form has a fundamental of 1,000,000 lines and a third harmonic of 100,000 lines. The flux densities, due to the fundamental and the third harmonic in the space distribution of the flux, have their maximum values at the center of the pole faces and both fields have the same directions at these points. The alternator is driven at a speed of 1800 r.p.m. (*a*) Calculate the Y-phase and terminal e.m.f.'s of the alternator when full-pitch coils are used. (*b*) Calculate the Y-phase and terminal e.m.f.'s, when the coils have $\frac{2}{3}\pi$ pitch. (*c*) What are the wave shapes of the e.m.f.'s obtained in *a* and *b*?

145. Prove that the armature reaction field due to

the armature currents in a polyphase alternator rotates at the same speed and in the same direction as the main revolving field produced by d.c. excitation.

146. Prove that the induced e.m.f. in an armature coil of a polyphase alternator due to the flux produced by the armature currents alone is 90 time degrees behind the current in the coil. (The positive sense of this e.m.f. is assumed the same as the positive sense of the current in the coil.) What other assumptions did you make in your proof? Why?

147. How are alternators rated? (See A.I.E.E. Standardization Rules.)

148. A single-phase alternator with a distributed mesh armature winding is rated at 5 kv-a., 110 volts. What would be the kv-a. and voltage rating of this machine if connected for three-phase operation? (Assume the same armature copper loss in each case.)

149. Define: (*a*) armature phase effective resistance; (*b*) armature phase leakage reactance; (*c*) armature phase reaction electromotive force; (*d*) synchronous reactance; (*e*) synchronous impedance.

150. How is the synchronous impedance of an alternator measured (see A.I.E.E. Standardization Rules)? In what way does the synchronous impedance of an alternator depend on the field current?

151. Why is the ratio of the effective resistance of an alternator winding to its d.c. resistance larger than a similar ratio for a transformer?

152. (*a*) Explain the meaning of the following equation for the terminal voltage of an alternator:

$$\dot{V} = \dot{E} - \dot{Z}\dot{I}$$

where V = the terminal voltage, the resultant rise in voltage through the armature winding taken positive in the same sense in which the armature current is taken as positive;

E = the induced e.m.f. due to the field flux produced by the field current alone (no armature current) taken positive in the same direction in which the armature current is taken as positive;

ZI = the voltage drop due to resistance, leakage reactance, and armature reaction taken positive in the same direction in which the armature current is taken as positive;

$\dot{Z}\dot{I} = r\dot{I} + jx\dot{I} + j\,\dot{E}_{ar}$;

rI = the effective resistance drop;

xI = the leakage reactance drop;

E_{ar} = the voltage drop due to the e.m.f. produced by the armature reaction field alone.

The above voltages and currents may be expressed in Y-phase or delta-phase values.

(*b*) Construct a complete vector diagram showing the relations between the e.m.f.'s, the voltage drops, and the

current in an armature phase of an alternator when the alternator delivers rated kv-a. at rated voltage to, (1) a zero lagging power-factor load, (2) a zero leading power-factor load, and (3) a unity power-factor load.

153. A single-phase alternator delivers 4 kv-a. at 120 volts to, (*a*) a leading power-factor load, (*b*) a unity power-factor load, and (*c*) a lagging power-factor load. The field currents required to maintain 120 volts for the different loads were 0.7, 1.1, and 1.2 amperes respectively. Explain.

154. A certain alternator delivers rated kv-a. at rated voltage to, (*a*) a highly inductive load, and (*b*) a highly condensive load. For which load are the losses in the alternator the largest? Explain.

155. The following things are known about a 5000-kv-a., 6700-volt, 60-cycle, three-phase synchronous generator:

Effective resistance of armature at 75° C. = 0.12 ohm per Y-phase.

Synchronous impedance of armature at 75° C. = 1.8 ohms per Y-phase.

The generator delivers rated kv-a. at rated voltage and at, (*a*) 80 per cent lagging power-factor, (*b*) unity power-factor, and at (*c*) 80 per cent leading power-factor. (1) Calculate the Y-phase excitation voltage required for each load. The excitation voltage is the induced voltage due to the flux produced by the field current alone (no armature current). (2) Construct complete vector diagrams showing the voltage and current relations for each load.

156. The following data were obtained on a 15-kv-a., 220-volt, 60-cycle, three-phase generator:

No-Load Saturation Curve

Frequency	Field Current. Amps.	Terminal Voltage, Volts
60	2	80
60	4	146
60	6	198
60	10	264
60	12	284
60	14	296
60	16	304

Zero Power-Factor Saturation Curve, Armature Current 39.4 Amps., Temperature 75° C.

Frequency	Field Current, Amps.	Terminal Voltage, Volts
60	6.9	0
60	8.0	40
60	10.0	103
60	12.0	158
60	14.0	199
60	16.0	229
60	18.0	251

D.C. resistance measured between armature terminals referred to 75° C. = 0.260.

Ratio $\frac{R_{a.c.}}{R_{d.c.}} = 1.34$, at 75° C.

(*a*) Plot the no-load and the zero power-factor curves and paste them in the book for future reference.

(*b*) Calculate: (1) the effective Y-phase resistance; (2) the Y-phase synchronous impedance at rated terminal voltage, rated current, and at zero lagging power-factor; and (3) the Y-phase synchronous reactance at rated terminal voltage, rated current, and at zero lagging power-factor.

157. The generator mentioned in Problem 156 supplies rated kv-a. at rated voltage to an 87.4 per cent lagging power-factor load. Calculate the voltage regulation of the alternator at 75° C. Use one of the methods outlined in the A.I.E.E. Standardization Rules.

158. Repeat the calculations in Problem 157 for a unity power-factor load.

159. Repeat the calculations in Problem 157 for an 87.4 per cent leading power-factor load.

160. Repeat the solution of Problem 157, using the e.m.f. and m.m.f. methods. (These methods are now obsolete but can be found explained in early textbooks.) Why are these old methods fundamentally wrong?

161. Estimate the zero power-factor curve in Problem 156. Compare your curve with that found by test. Why are the curves of slightly different shape? See A.I.E.E. Rules.

162. Two identical 6500-kv-a., 12,000-volt, 25-cycle, three-phase, Y-connected alternators are connected in parallel and supply 10,000 kv-a. at a terminal voltage of 12,000 volts to a 91 per cent lagging power-factor load. The following things are known about the alternators:

No-Load Saturation Curve at 25 Cycles

Terminal Voltage	Field Current	Terminal Voltage	Field Current
2000	30	10,000	160
4000	60	12,000	210
6000	90	14,000	270
8000	120	16,000	360

Effective resistance per Y-phase referred to 75° C. = 1.73 ohms.

Zero Power-Factor Curve Referred to 75° C. (Line Current 313 Amperes, Frequency 25 Cycles)

Terminal Voltage	Field Current	Terminal Voltage	Field Current
0	100	8,000	230
2000	125	10,000	280
4000	160	12,000	340
6000	190	14,000	425

1. (*a*) Plot the no-load saturation and zero power-factor curves; paste the curves in the book for future reference. (*b*) Calculate the Y-phase synchronous impedance at rated voltage and current.

2. One of the alternators operates at a field current of 350 amperes and delivers 4000 kilowatts. (*a*) Calculate the field current required on the other alternator to maintain 12,000 volts, at 25 cycles at the load. (*b*) Calculate the power-factor at which each alternator operates. (*c*) Draw a complete vector diagram showing

all current and voltage relations. (*d*) What adjustments are necessary to make the alternators share the loads equally?

163. The alternators specified in Problem 162 are operated at no-load (no external load connected to the bus-bars). The voltage between the bus-bars is 12,000 volts and the frequency is 25 cycles. The excitation voltage to neutral in a phase of one of the machines is 7500 volts and leads the bus-bar voltage to neutral in this phase by an angle of 10 time degrees. (*a*) Calculate the circulating current. (*b*) Calculate the electrical power which is transferred from one of the machines to the other.

164. Two 2300-volt, three-phase, Y-connected alternators are connected in parallel and supply an 824-kv-a. load at 2300 volts and 95 per cent leading power-factor. The ratings and the constants of the machines are as follows:

Rating	Y-phase Effective Resistance	Y-phase Synchronous Impedance
750 kv-a.	0.141 ohm	2.31 ohm
500 kv-a.	0.220 ohm	2.50 ohm

The governors of the prime movers and the field excitations of the generators are so adjusted that each machine operates at the same power-factor and delivers a current in direct proportion to its kv-a. rating. (*a*) Draw a complete vector diagram showing the voltage and current relations. (*b*) Calculate the excitation e.m.f.'s required and the phase angle between them.

165. Two alternators, A and B, are operated in parallel.

(*a*) What factors determine the power output of A?

(*b*) What adjustments are necessary to make the circulating current zero? How is it possible to tell when the circulating current is zero?

(*c*) How are the current outputs of the machines brought in phase with the load current? How can one tell when this condition is satisfied?

(*d*) Under what conditions are the outputs of the alternators inversely proportional to the synchronous impedances of their armatures?

166. Show that the power-factor of an alternator operating in parallel with others becomes leading when its field excitation is lowered sufficiently.

167. Show that the power-factor of an alternator operating in parallel with others becomes lagging when its excitation is raised beyond a certain value.

168. The governor on the prime mover of an alternator operating in parallel with others of much larger capacity is set at a definite point. Why is it true that the output of this alternator cannot be changed (neglecting the change in copper loss in its armature) by changing the field current?

169. Two identical 2300-volt, Y-connected alternators are operated in parallel. Each alternator is rated at 750 kv-a., has a Y-phase effective resistance of 0.123 ohm, and a Y-phase synchronous impedance of 2.13 ohms. The Y-phase excitation e.m.f. of each alternator is 1330 volts. The excitation e.m.f.'s in corresponding phases are 5 time degrees out of phase (with reference to the bus-bars). No external load is connected to the bus-bars. Calculate the power tending

to make the phase-angle between the excitation e.m.f.'s zero.

170. The following things are known about two 10,000-kv-a., 4150-volt, 60-cycle, three-phase, Y-connected generators:

Alternator	Y-phase Effective Resistance at 75° C.	Y-phase Synchronous Impedance at 75° C.
A	0.0062 ohm	0.682 ohm
B	0.0091 ohm	0.682 ohm

The two generators are operated in parallel and supply a balanced three-phase load of 15,000 kv-a. at 4150 volts. The governors on the prime movers and the excitation voltages on the generators are so adjusted that the copper loss in the armatures is a minimum. Neither of the machines is allowed to carry an overload. Calculate: (*a*) the current output of each generator; (*b*) the power output of each generator; and (*c*) the excitation voltage required on each generator and the phase-angle between the excitation voltages in corresponding phases.

171. A three-phase synchronous generator is rated at 850 kv-a. at 11,000 volts. The losses in the generator at rated unity power-factor load are:

Stator copper loss referred to 75° C..... 7.81 kw.
Rotor copper loss referred to 75° C..... 5.30 kw.
Core loss..... 20.3 kw.
Friction and windage loss..... 12.0 kw.

The Y-phase synchronous impedance (referred to 75° C.) of the generator is 42 ohms. The operating

temperature of the machine is 75° C. Calculate the efficiency of the generator at, (*a*) rated unity power-factor load, and (*b*) at one-half rated armature current, unity power-factor load. (Assume that the field current of the generator is constant and that the temperature of the machine is also constant.)

172. (*a*) Show by a series of diagrams that when a two-phase current is established in a two-phase armature winding a revolving magnetic field is produced. (*b*) Show that the speed of this revolving field in revolutions per second is $\frac{2f}{p}$, where f is the frequency of the current, and p the number of poles for which the armature is wound.

173. Explain in detail the meaning of the following relations for the applied voltage to a synchronous motor:

$$\dot{V} = \dot{E}' + \dot{Z}\dot{I} = \dot{E}' + (r + jX)\dot{I},$$

$$\dot{V} = \dot{E}'' + (r + jx)\dot{I},$$

$$\dot{V} = \dot{E}''' + r\dot{I},$$

where V = the total voltage drop per Y-phase (assumed positive in the same direction in which the current is taken positive);

E' = the voltage drop per Y-phase (assumed positive in the same direction in which the current is taken positive) due to the induced e.m.f. caused by the flux produced by the field current alone;

E'' = the voltage drop per Y-phase due to the induced e.m.f. caused by the mutual flux produced by the armature currents and field current combined;

E''' = the voltage drop per Y-phase due to the e.m.f. produced by all the flux linking a phase;

r = effective Y-phase resistance;

X = Y-phase synchronous reactance;

x = Y-phase leakage reactance;

and

I = Y-phase current.

174. Construct a complete voltage drop-current vector diagram for a synchronous motor, showing all the quantities given by the equations in Problem 173.

175. The following things are known about a 10-h.p., 220-volt, 6-pole, 60-cycle, three-phase synchronous motor:

Effective resistance per delta-phase at the operating temperature, 0.119 ohm;

Synchronous impedance per delta-phase at the operating temperature, 0.785 ohm.

The motor is connected to a 220-volt, 60-cycle, balanced three-phase supply, and the current per terminal is 21 amperes. The field current of the motor is so adjusted that the excitation voltage is 220 volts. (1) Calculate: (*a*) the speed of the motor; (*b*) the power-factor at which the motor operates; and (*c*) the total power which leaves the armature winding of the motor. (2) Construct a complete voltage drop-current vector diagram showing the applied voltage, the voltage drop

due to the e.m.f. induced by the flux established by the field current alone, the voltage drop due to the synchronous impedance, and the phase current.

176. The following things are known about a 12-h.p., 220-volt, three-phase, 60-cycle, Y-connected synchronous motor:

No-Load Saturation Curve (Frequency 60 Cycles)

Terminal Voltage, Volts	Field Current, Amps.	Terminal Voltage, Volts	Field Current, Amps.
146	4.0	264	10.0
199	6.0	284	12.0
238	8.0	296	14.0

Zero Power-Factor Curve (Frequency 60 Cycles, Temperature 25° C. and Line Current 39.4 Amperes)

Terminal Voltage, Volts	Field Current, Amps.	Terminal Voltage, Volts	Field Current, Amps.
0	6.9	199	14.0
158	12.0	229	16.0

D.C. resistance measured between terminals at 25° C. = 0.253 ohm;

Effective resistance = 1.35 times d.c. resistance at 75° C.;

Total friction + windage + core loss = 1.95 kw. (Assume that this loss is constant at all loads.)

The motor is connected to a balanced 220-volt, 60-cycle, three-phase supply. The motor delivers 8 h.p. at the pulley and operates at unity power-factor and a temperature of 75° C.

(*a*) Plot the no-load saturation and zero power-factor curves on the same sheet of paper and paste them in the book. (*b*) Calculate the field current required on the motor. (The motor is excited by an entirely separate machine.)

177. The following things are known about a 1070-h.p., 11,000-volt, 60-cycle, three-phase synchronous motor:

No-Load Saturation Curve (Frequency 60 Cycles)

Terminal Voltage, Volts	Field Current, Amps.	Terminal Voltage, Volts	Field Current, Amps.
4,000	20	12,300	80
8,100	40	14,150	120
10,800	60	14,750	140

Zero Power-Factor Curve (Frequency 60 Cycles, Current 44.6 Amperes, Temperature 25° C.)

Terminal Voltage, Volts	Field Current, Amps.	Terminal Voltage, Volts	Field Current, Amps.
0	43	10,600	120
6800	80	11,800	140
9100	100	12,400	160

D.C. resistance measured between terminals at 25° C. = 2.64 ohms.

$\frac{R_{a.c.}}{R_{d.c.}} = 1.27$;

Core loss, 8.2 kw.;

Friction and windage loss, 9.3 kw.

The rotor of the motor is excited from an entirely separate unit. The motor delivers 800 h.p. and is supplied by power from a 11,000-volt, 60-cycle, balanced three-phase supply.

(*a*) Plot the no-load and zero power-factor curves and paste them in the book. (*b*) Plot a V-curve for the motor corresponding to an output of 800 h.p. The excitation voltages may be calculated analytically or may be obtained from a circle diagram. (*c*) What assumptions did you make in (*b*)?

178. The synchronous motor specified in Problem 176 is operated from a balanced 220-volt, 60-cycle, three-phase supply. The motor is excited from a separate unit, delivers 5 h.p., and operates at unity power-factor. Calculate: (*a*) the field current required on the motor; (*b*) the power-factor of the motor when the frequency and the field current are the same as in (*a*) but the applied voltage is 280 volts.

179. The synchronous motor specified in Problem 176 is operated in parallel with a 10-h.p. induction motor. The input to the induction motor is 11.6 kv-a. at a lagging power-factor of 83 per cent. The output of the synchronous motor is 5 h.p. The line voltage is 220 volts and the frequency 60 cycles. The field current of the synchronous motor is so adjusted that the combined load operates at unity power-factor. (1) Calculate: (*a*) the current input to the induction

motor; (*b*) the current input to the synchronous motor; (*c*) the excitation voltage and the field current of the synchronous motor; (*d*) the current input to the synchronous motor; and (*e*) the combined current of the two loads. (2) Draw a complete voltage drop-current vector diagram showing the Y-phase applied voltage, the terminal current of the induction motor, the terminal current of the synchronous motor, and the total line current. (3) What assumptions have you made in the solution of this problem?

180. The synchronous motor in Problem 176 is connected to a 220-volt, 60-cycle, balanced three-phase supply. It is now operated at full-load. (*a*) Calculate the maximum and minimum theoretical excitation voltages at which the motor will operate. (*b*) Do you consider these voltages practical? Why? (*c*) What assumptions have you made in the solution of this problem?

181. The synchronous motor in Problem 176 is connected to a 220-volt, 60-cycle, balanced three-phase supply. The motor is loaded until it fails. (1) Calculate: (*a*) the maximum theoretical torque; (*b*) the theoretical line current when the motor fails; and (*c*) the corresponding power-factor. (2) Explain why the values calculated in (1) can probably not be checked in the laboratory?

182. The motor in Problem 177 is connected to a 11,000-volt, 60-cycle, balanced three-phase supply. The motor supplies a mechanical load of 600 h.p. (1) Calculate: (*a*) the maximum theoretical leading power-factor angle at which the motor can be operated without exceeding its current rating; (*b*) the maximum theoretical lagging power-factor angle; and (*c*) the

field currents required to obtain these angles. (2) What assumptions did you make in (1)?

183. The motor specified in Problem 177 is connected to a 11,000-volt, 60-cycle, balanced three-phase supply. The field current is adjusted for an excitation e.m.f. of 125 per cent of the rated terminal voltage of the motor. (*a*) Calculate the maximum theoretical motor power possible for this value of excitation. (*b*) Calculate the maximum theoretical motor power when the excitation voltage is 75 per cent of the rated terminal voltage of the motor. (*c*) What assumptions have you made in the solution of this problem?

184. A certain synchronous motor is operated at constant field current. Prove that the maximum torque developed by the motor is a function of the first power of the applied terminal voltage.

185. A certain synchronous motor is operated at constant power-factor. Prove that the maximum torque is a function of the square of the voltage.

186. A three-phase synchronous motor is operated in parallel with a group of three-phase induction motors. The line voltage is 440 volts. The induction motors require a combined delta-phase current of 400 amperes, and operate at a lagging power-factor of 83 per cent. The total power input to the synchronous motor is 25 kilowatts. Calculate the kv-a. rating of the synchronous motor required to make the power-factor of the combined load unity.

187. The synchronous motor specified in Problem 177 is supplied by power from a 11,000-volt, 60-cycle, balanced, three-phase power supply. The motor delivers 1070 h.p. The field is over-excited, so that the motor armature terminal current is 60 amperes.

The resistance of the rotor field winding is 0.81 ohm at 75° C. The motor operates at a temperature of 75° C. (1) Calculate: (*a*) the power-factor; (*b*) the rotor field loss; and (*c*) the efficiency of the motor at 75° C. (2) The field current is changed so that the motor operates at unity power-factor. Calculate the efficiency at 75° C.

188. The synchronous motor specified in Problem 176 is operated at the end of a three-phase transmission line. The motor is driven by an alternator whose constants are equal to those of the motor. The fields of the two machines are excited from an entirely separate unit. The field current in each machine is 10 amperes. The frequency is 60 cycles. The transmission line has a resistance of 0.42 ohm and a reactance of 0.5 ohm. The load on the synchronous motor is 5 h.p. The machines operate at a temperature of 25° C. (1) Calculate: (*a*) the voltage across the terminals of the motor; (*b*) the voltage across the terminals of the generator; and (*c*) the phase-angle between the excitation e.m.f.'s of the two machines. (2) Construct a complete vector diagram showing the voltage and current relations.

189. Draw an armature wiring diagram of a 4-pole, six-phase, six-ring, synchronous converter, showing the proper connections to the six rings and the proper location of the d.c. brushes. A ring winding, although not used in practice, may be used in illustrating the connections.

190. Why is a closed-circuit winding (a mesh winding) essential on a synchronous converter?

191. Derive a formula showing the relation between

the a.c. ring current and the total d.c. output current of a synchronous converter.

192. Derive a formula showing the relation of the a.c. voltage between adjacent rings to the d.c. voltage of a synchronous converter.

193. Derive a formula showing the relation between the a.c. component of the coil current to the d.c. component for a given coil on a synchronous converter.

194. Derive an equation for the resultant instantaneous current in a coil located on the armature of a synchronous converter.

195. A 4-pole, six-phase converter delivers 720 kilowatts on the d.c. side. The converter operates at a leading power-factor of 97 per cent, at an efficiency of 94 per cent, and at a d.c. voltage of 550 volts. (*a*) Calculate the r.m.s. value of the voltage between adjacent rings. (*b*) Calculate the r.m.s. value of the total current supplied to a ring. (*c*) Of what value are these calculations?

196. A 4-pole, three-phase converter delivers 425 kilowatts at a d.c. voltage of 250 volts. The converter operates at an efficiency of 90 per cent, and at a lagging power-factor of 87 per cent. (*a*) Plot the waves for the a.c. and d.c. components of the coil current in a coil located midway between the two tapping points of a phase. (*b*) Repeat (*a*) for coils located near the leading and trailing tapping points. (*c*) Plot the resultant current waves for the coils in (*a*) and (*b*).

197. The following things are known about a 2000-kw., 16-pole, 600-volt (d.c.), six-phase, 60-cycle, synchronous converter:

Losses measured at rated unity power-factor load (temperature 25° C.)

Armature copper loss	14.7 kw.
Series field loss	3.2 kw.
Commutating field loss	3.75 kw.
Shunt field loss (including rheostat)	5.9 kw.
Core loss	15.6 kw.
Total friction loss	30.0 kw.

The field on the converter is adjusted so that unity power-factor is obtained on the a.c. side. The converter operates at a temperature of 25° C. Calculate the efficiency at, (*a*) 150 per cent rated load, (*b*) rated load, and (*c*) 50 per cent rated load.

198. What will be the voltage and current ratings of the secondary windings of three transformers required to supply the converter specified in Problem 197 when, (*a*) the transformers are diametrically connected, and (*b*) when the transformers are connected in double-delta? The converter is assumed to operate at rated d.c. voltage and at unity power-factor on the a.c. side.

199. A synchronous converter like that specified in Problem 197 is supplied from a 213-volt, 60-cycle, six-phase source. Reactors having effective resistances of 0.0001 ohm and inductive reactances of 0.08 ohm are connected in each of the six phases. The load and excitation on the converter are so adjusted that the current in the six-phase leads is 1100 amperes. The power-factor of the converter measured at the converter rings is 0.90 leading. Calculate: (*a*) the d.c. voltage with the reactors in the circuit; and (*b*) the d.c. voltage with the reactors short-circuited.

200. Give reasons for using speed-limiting devices on synchronous converters.

201. What would be the approximate rating of the machine specified in Problem 197 if it were operated as an ordinary shunt generator?

202. The primary windings of transformers supplying a six-phase, split-pole converter must not be connected in delta when the secondaries are connected diametrically. Explain.

203. Under what conditions is the d.c. voltage of a synchronous converter independent of the field current? Explain fully.

204. For ordinary operating conditions, the e.m.f. induced in the armature of a synchronous converter due to the armature reaction field is small. Explain fully.

205. For what conditions of operation is the armature reaction field of a converter large? Explain fully.

206. Explain in detail the following:

(*a*) Synchronous motors, for most satisfactory operation, must be provided with salient poles.

(*b*) Induction motors, for most satisfactory operation, must be provided with smooth cylindrical rotors.

(*c*) An ordinary induction motor (a motor with no special equipment) must operate at less than synchronous speed.

(*d*) In a synchronous motor the rotor is polarized by d.c. The current is conducted to the rotor winding.

(*e*) In an induction motor the rotor is polarized by a.c. The current is induced in the rotor winding.

(*f*) The rotor poles (the poles produced by the rotor currents alone) of an induction motor continuously

slip forward (in the direction of rotation) relative to the rotor laminations.

207. Show analytically that the frequency of the induced current in any phase of the rotor of an induction motor is equal to sf_1, where s is the slip and f_1 the frequency of the current in the stator winding.

208. Show analytically that the rotor field (the field due to the rotor currents alone) travels at the same speed and in the same direction as the field produced by the stator currents alone.

209. The stator and rotor windings of an induction motor are Y-connected. The turn ratio is 1. Show that the induced e.m.f. (the e.m.f. due to the air-gap flux) in a rotor phase is equal to the product of the slip by the induced e.m.f. in a stator phase.

210. Show that the reactance of a rotor phase of an induction motor is directly proportional to the slip.

211. Show that the power, $n\ (s\ E_2 I_2) \cos \theta_2$, is equal to the total rotor copper loss, where n is the number of rotor phases, s the slip, E_2 the induced e.m.f. in a rotor phase when the rotor is locked and open-circuited, I_2 the rotor current, and $\cos \theta_2$ the power-factor of the rotor. Note that the power calculated by this formula is the power necessary to excite the rotor.

212. Show that the power $n\ (1 - s)\ E_2 I_2 \cos \theta_2 = \frac{Fv\ 746}{550}$, where F is the force exerted on the rotor in pounds, and v the velocity of a point on the surface of the rotor in feet per second.

213. Prove that an induction motor operating at a slip s and with the rotor short-circuited, takes the same current from the supply and at the same power-factor as an identical motor connected to the

same supply but with the rotor locked and resistances equal to $r_2\left(\frac{1-s}{s}\right)$ inserted in each rotor phase, where r_2 is the rotor phase resistance.

214. Based on your proof in Problem 213, show that the horse power developed in the rotor of an induction motor equals $\frac{n\, r_2}{746}\left(\frac{1-s}{s}\right) I_2{}^2$.

215. Two identical polyphase induction motors are connected to the same power supply. One of the motors is operated at a slip of 10 per cent and with its rotor winding short-circuited. The other motor is locked and a resistance $9r_2$ is added externally to each rotor phase (r_2 is the rotor phase resistance). Determine the relative values of: (*a*) the rotor e.m.f.'s; (*b*) the rotor currents; (*c*) the power-factors of the rotor circuits; (*d*) the stator currents; (*e*) the stator power-factors; (*e*) the power transferred across the air-gaps to the rotor iron; (*f*) the power losses in the rotors; and (*g*) the power loss in the resistances ($9r_2$) and the total mechanical power developed in the rotor of the revolving machine.

216. The stator of a certain 60-cycle polyphase induction motor is wound for 8 poles. Calculate: (*a*) the speed of the revolving field in revolutions per minute; (*b*) the number of magnet poles formed on the rotor; (*c*) the frequency of the currents induced in the rotor at standstill; (*d*) the frequency of the currents in the rotor when the slip is 10 per cent; and (*e*) the speed of the rotor when the slip is 10 per cent.

217. (*a*) Show that, per ampere of current in the stator and rotor phases of an induction motor, the maximum torque is produced when the centers of the

poles formed by the rotor currents alone are 90 electrical space degrees displaced from the centers of the poles formed by the stator currents alone. (*b*) Show that in an ordinary induction motor (a motor with no special equipment) the centers of the poles referred to in (*a*) cannot be 90 electrical space degrees apart.

218. (*a*) Show that the insertion of resistance in the rotor circuit of an induction motor has a tendency to make the angle between the centers of the stator and rotor poles (the poles referred to in Problem 217), 90 electrical space degrees. (*b*) Show that the insertion of inductance in the rotor phases tends to make the centers of the stator and rotor poles coincide.

219. Figure 13 represents an experimental squirrel-cage induction motor.

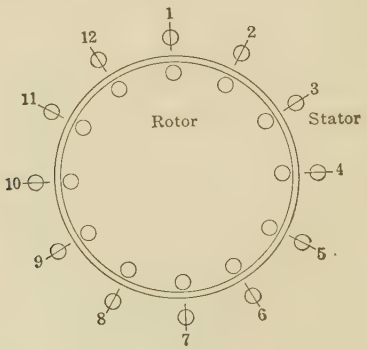


Fig. 13.

(*a*) Show how to connect the stator conductors (1, 2, 3, . . . 12) so as to obtain a three-phase, 4-pole, Y-connected motor. Connect the terminals of the motor to three supply mains marked *A*, *B*, and *C*.

of a three-phase transmission line. Mark the common connecting point of the phases O.

(*b*) Indicate the directions of the currents in the stator conductors, and show the location of the centers of the poles due to the stator currents alone. Assume an instant when the current from $A - O$ has its maximum value. Assume that the phase rotation of the line currents is $AO - BO - CO$.

(*c*) Indicate the direction of rotation of the revolving field produced by the stator currents in (*b*).

(*b*) Indicate the direction of the rotor currents induced by the revolving field. Mark the approximate positions of the centers of the poles produced by the rotor currents alone.

220. A certain induction motor operating at no-load requires a current of about 25 per cent of the rated current of the motor. (*a*) Why is the no-load current of an induction motor high? (*b*) Is the presence of this high no-load current objectionable to, (1) the consumer, and (2) the central station?

221. Show that the magnetic reaction produced by the rotor currents of an induction motor is identical in every way with the magnetic reaction produced by the secondary currents in an ordinary power transformer.

222. Show that the applied voltage to a stator phase of an induction motor may be expressed by the equation:

$$\dot{V}_1 = \dot{E}_1 + \dot{Z}_1\dot{I}_1 = \dot{E}_1 + (r_1 + jx_1)\,\dot{I}_1$$

where E_1 = voltage drop due to the induced e.m.f. in a stator phase caused by the air-gap flux;

r_1 = effective resistance per stator phase;

x_1 = leakage reactance per stator phase;

and

I_1 = the current per stator phase.

223. Outline the experimental tests necessary for measuring the constants of an induction motor. Give a wiring diagram for each test and show how the instruments should be arranged to obtain the most reliable results.

224. The following measurements were made on a 3-horse power, 4-pole, three-phase, 110-volt, 60-cycle induction motor:

D.C. resistance between stator terminals at 26° C. = 0.334 ohm.

D.C. resistance between rotor terminals (the machine has a three-phase rotor) at 26° C. = 0.52.

Voltage ratio test (*rotor stationary and open-circuited*)

Stator voltage between terminals, 98.

Rotor voltage between terminals, 108.

No-load test (*rotor running free and short-circuited, temperature* 26° *C.*)

Terminal voltage, 110 volts.

Frequency, 60 cycles.

Total power input, 274 watts (170 watts core loss and 104 watts friction and windage).

Impedance test (*rotor short-circuited and locked, temperature* 26° *C.*)

Terminal voltage, 17.6.

Frequency, 60 cycles.

Current per terminal, 12.5 amperes.

Total power input (three phases), 244 watts.

(*a*) Calculate the Y-phase constants of the motor referred to 75° C. Assume that the secondary (rotor) leakage reactance referred to the primary (stator) is the same as the primary reactance.

(*b*) Calculate the delta-phase constants of the motor referred to 75° C.

(*c*) Plot a curve showing the relation between torque and slip. The rotor is short-circuited and the temperature of the machine is 75° C. Express the torque in pound-feet and the slip in per cent. What assumptions have you made?

(*d*) Calculate the maximum running torque and the slip at which it occurs. The answer to this question may be obtained from the curve plotted in (*c*).

(*e*) Calculate the starting torque (rotor short-circuited). The answer to this question may also be obtained from the curve plotted in (*c*).

(*f*) Calculate the value of the resistance which must be added to each rotor phase to obtain the maximum torque at starting.

(*g*) Plot the torque-slip curve when the total rotor resistance is such as to obtain maximum torque at starting. Express the torque in pound-feet and the slip in per cent.

(*h*) Plot a curve showing the relation between efficiency and horse-power output for, (1) the rotor short-circuited, and (2) resistances added to each rotor phase to obtain maximum torque at starting.

225. (*a*) Show that the voltage drop relation

$$\dot{V} = \dot{E} + \dot{Z}\dot{I}$$

may be represented by a circle diagram, provided V and Z have constant values.

(*b*) Show that the current relation $\dot{I} = \frac{\dot{V} - \dot{E}}{\dot{Z}}$ may be represented by a circle diagram, provided V and Z have constant values.

Note that the relations (*a*) and (*b*) are the fundamental voltage and current relations for an a.c. motor.

(*c*) Prove that when the values in equations (*a*) and (*b*) are for the same circuit, the diameter of the current circle lags behind the diameter of the voltage-drop circle by an angle whose tangent is $\frac{x}{r}$.

226. A coil having a constant resistance of 1 ohm and a constant inductive reactance of 2 ohms is connected in series with a variable resistor to a 110-volt, 60-cycle supply. (*a*) Construct a voltage-drop circle diagram. To obtain the positions of the points, construct voltage-drop vector diagrams for different values of added resistance. Assume that the vector which represents the applied voltage as fixed in position. (*b*) Construct the current circle diagram.

227. Compare the primary circuit of an induction motor with the series circuit in Problem 226. Under what conditions may a circle diagram be used in solving induction motor problems?

228. The following things are known about a 1000-horse power, 2200-volt, 25-cycle, 4-pole, three-phase induction motor:

Ratio of primary turns to the secondary turns 1.6 (the primary and secondary windings are both Y-connected);

D.C. resistance of the rotor, measured between terminals and referred to 75° C. = 0.0668 ohm;

$\frac{R_{a.c.}}{R_{d.c.}} = 1.25$ (at 25 cycles.)

No-load test performed at rated voltage and at rated frequency

No-load terminal current, 41 amperes;
Total core loss, 11,500 watts;
Total friction and windage loss, 14,400 watts.

Locked test performed at rated frequency. (The values given are referred to rated voltage and to a temperature of 75° C.):

Terminal voltage, 2200 volts;
Terminal current, 1840 amperes;
Total power input, 2036 kilowatts.

(1) Construct a complete circle diagram and from the diagram, for rated output of the motor determine, (*a*) the terminal current, (*b*) the power-factor, (*c*) the efficiency, (*d*) the speed of the motor in r.p.m., and (*e*) the torque developed by the motor expressed in pound-feet.

(2) From the circle diagram determine, (*a*) the maximum power-factor and the corresponding output, (*b*) the maximum possible efficiency, (*c*) the maximum possible torque, and (*d*) the torque at starting.

229. The following things are known about a 10-horse power, 440-volt, 25-cycle, three-phase, 6-pole induction motor:

Ratio of primary to secondary turns 3.47 (the primary and secondary windings are Y-connected);

D.C. resistance of primary, 0.71 ohm per Y-phase;
D.C. resistance of secondary, 0.084 ohm per Y-phase;
$\frac{R_{\text{a.c.}}}{R_{\text{d.c.}}} = 1.26$ (at 25 cycles);
Primary leakage reactance 1.76 ohm per Y-phase;
Secondary leakage reactance referred to the primary winding, 1.81 ohms;
No-load current at rated voltage and frequency, 9.6 amperes;
Total power loss at no-load 636 watts (core loss 480 watts; friction and windage 156 watts).

When the no-load voltage is 125 per cent of rated voltage, the no-load current is 141 per cent of the normal value, and the no-load loss is 132 per cent of the normal value.

(1) The motor is supplied at rated voltage and frequency and delivers rated horse power. Calculate: (*a*) the speed in r.p.m.; (*b*) the total primary current; and (*c*) the primary power-factor.

(2) The motor is supplied at 125 per cent rated voltage, at rated frequency, and delivers rated horse power. Calculate: (*a*) the speed in r.p.m.; (*b*) the primary current; and (*c*) the primary power-factor.

230. The following things are known about a 25-horse power, 220-volt, 60-cycle, three-phase induction motor:

Primary d.c. resistance per Y-phase, 0.094 ohm;
Secondary d.c. resistance per Y-phase referred to primary turns, 0.102 ohm;
No-load current, 10.9 amperes;
Core loss, 585 watts;
Friction and windage loss, 221 watts.

Locked-test meter indications

Line voltage, 75 volts;
Line current, 56 amperes;
Total locked power, 2460 watts.

The above values are all referred to a temperature of 75° C.

The induction motor is operated in parallel with the synchronous motor specified in Problem 176. The mechanical load on the induction motor is 25 horse power and the load on the synchronous motor is 5 horse power.

(*a*) The line voltage is 220 volts and the frequency is 60 cycles. Calculate the field current required on the synchronous motor to make the power-factor of the combined load unity.

(*b*) The line voltage is now raised to 125 per cent of the rated voltage of the machines. The frequency is 60 cycles. The synchronous motor field current is the same as in (*a*). Calculate the power-factor of the combined load. Assume that a 25 per cent increase in the applied voltage to the induction motor causes its no-load current to increase 45 per cent and its no-load loss to increase 27 per cent.

231. Prove that the addition of resistance to the rotor circuit of an induction motor does not always increase the starting torque.

232. Two induction motors A and B are operated in concatenation. The stator winding of A is connected to the supply and the rotor winding of A is connected to the stator winding of B. The rotor winding of B is short-circuited. (*a*) Show that the conditions for torque are best in both machines when the set operates

near synchronous speed of B. (*b*) Show that synchronous speed of B depends on the primary frequency of A, the slip of A, and the number of poles on B. (*c*) Derive a formula for the approximate running speed of the set.

233. Motor A in Problem 232 is wound for 12 poles and B for 8 poles. The frequency of the supply is 25 cycles. (*a*) Calculate the approximate no-load speed when motor A is connected to the supply and its rotor short-circuited. (*b*) Calculate the approximate no-load speed when motor B is connected to the supply and its rotor short-circuited. (*c*) The stator of A is connected to the supply and its rotor is connected to the stator of B. The rotor of B is short-circuited. Calculate the no-load speed of the set. (*d*) The stator of B is connected to the supply and the rotor of B is connected to the stator of A. The rotor of A is short-circuited. Calculate the approximate no-load speed of the set.

234. A 10-horse power, 4-pole, 220-volt, 60-cycle induction motor and a 10-horse power, 6-pole, 220-volt, 60-cycle induction motor have a common shaft. The 4-pole motor is connected to a 220-volt, 60-cycle supply, the rotor of the 4-pole motor is connected to the stator of the 6-pole motor. The rotor of the 6-pole motor is short-circuited. Both motors have a turn ratio of unity. (*a*) Calculate the maximum safe horse power output of the combination. (*b*) What proportion of the load does each motor take in (*a*)?

235. Show that the insertion of the right amount of resistance in the rotor circuit of motor B in Problem 232 will improve the starting torque of both motors.

236. A 550-horse power, 2000-volt, 25-cycle, 32-pole, three-phase induction motor is operated at rated vol-

tage and frequency. The following things are known about the motor:

D.C. resistance of the primary per delta-phase at 25° C. = 0.456 ohm;

D.C. resistance of secondary per delta-phase at 25° C. = 0.0127 ohm;

$\frac{R_{a.c.}}{R_{d.c.}}$ for the primary winding, 1.51 (at 25 cycles);

Ratio of primary turns to secondary turns, 6.

Locked-test meter indication (*temperature* 25° C.)

Current, 225 amperes;

Voltage, 595 volts.

No-load test results

Magnetizing component of line current, 61 amperes;

Core loss, 3.1 kilowatts;

Friction and windage loss, 11.75 kilowatts;

Induced e.m.f. due to the air-gap flux, 1865 volts.

Calculate at rated horse power output: (*a*) the speed in r.p.m.; (*b*) the efficiency; and (*c*) the power-factor.

237. (*a*) Show that an induction generator cannot deliver power at unity power-factor.

(*b*) Show that the load connected to an induction generator must be capable of taking a leading current from the line.

(*c*) Show that when an induction generator is operated in parallel with synchronous generators of much larger capacity, the frequency of the system is determined by the speed of the synchronous generators.

238. An induction generator and a synchronous generator are operated in parallel and supply a synchronous motor. (*a*) What will happen if the synchronous

generator is disconnected? (*b*) What factors will determine the frequency in (*a*)? (*c*) What factors will determine the terminal voltage of the induction generator in (*a*)? The synchronous motor is assumed to have less capacity than the induction generator.

239. The induction motor specified in Problem 228 is operated as an induction generator at a terminal voltage of 2200 volts, at a terminal current of 194 amperes, and at a frequency of 60 cycles per second. The operating temperature of the machine is 75° C. Calculate: (*a*) the speed of the generator; (*b*) the kilowatt output of the generator; and (*c*) the efficiency of the generator.

240. The self-inductance of each Y-phase (alone) of a Y-connected induction motor is 0.25 henry. What will be the total self-inductance formed by two Y-phases connected in series (the remaining phase disconnected)? Two answers are necessary.

241. Show analytically that in a single-phase induction motor the stator flux induces two e.m.f.'s in the rotor, one have a frequency of $\left(f - n\frac{p}{2}\right)$ and the other $\left(f + n\frac{p}{2}\right)$, where f is the frequency of the stator current, n the number of revolutions per second of the rotor, and p the number of poles on the stator.

242. Figure 14 is an oscillogram of the current wave in a rotor phase of a single-phase induction motor. The oscillogram was taken when the speed of the rotor was two thirds of synchronous speed. Give reasons for the peculiar shape of the waves. Why are the sharp peaks present in the current wave?

243. The induction motor specified in Problem 228 is operated with one of the line wires disconnected.

(*a*) Calculate: (1) the theoretical single-phase horse power rating; (2) the efficiency and the speed of the motor at this rating; (*b*) Construct a circle diagram for the motor and determine: (1) the theoretical load at which the power-factor is a maximum; and (2) the theoretical maximum possible efficiency and the corresponding load.

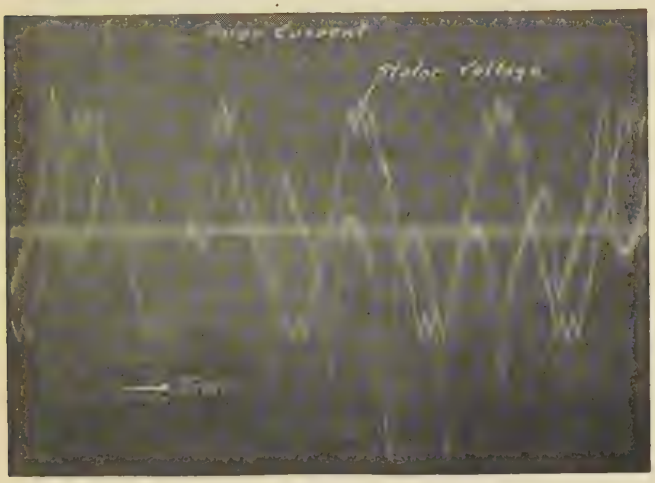


FIG. 14.

244. The following measurements were made on a 10-horse power, 110-volt, 25-cycle conductively compensated single-phase series motor:

Resistance test results

Effective resistance of the field winding, 0.041 ohm;
Effective resistance of the armature and compensating winding combined, 0.1160 ohm.

Armature locked-test results

Frequency, 25 cycles;
Line current, 105 amperes;
Voltage drop across main field, 45 volts;
Voltage across armature and compensating field combined, 14 volts.

The motor is operated at 110 volts, 25 cycles, and at a line current of 105 amperes. (*a*) Calculate: (1) the rotational e.m.f. induced in the armature; and (2) the power-factor at which the motor operates. (*b*) Construct a voltage drop-current vector diagram showing the voltage and current relations. (*c*) Construct a circle diagram for this motor.

245. Figure 15 represents a Winter-Eichberg-Latour compensated repulsion motor (see Pender's "Handbook for Electrical Engineers," 1922 edition, page 1023). Construct a complete voltage drop-current vector diagram showing how power-factor compensation is obtained. Show on your diagram the following quantities:

(*a*) Supply current, assumed positive in the direction indicated.

(*b*) Voltage drop, *A*–*B*, taken positive in the sense *A* to *B*.

(*c*) Rotational e.m.f. and transformer e.m.f., *C*–*D*, taken positive in the sense *C* to D.

(*d*) Rotational e.m.f. and transformer e.m.f., *E*–*F*, taken positive in the sense *E* to *F*.

(*e*) Voltage drop, *B*–*G*, taken positive in the sense *B* to *G*.

(*f*) Voltage drop across the supply, taken positive in the sense *A* to *G*.

In answering parts *c* and *d* assume the rotation of the armature as indicated and assume that the armature is so wound that a current entering brush *C* and leaving brush *D* will produce an armature field down. A current entering brush *E* and leaving brush *F* will then produce an armature field to the left.

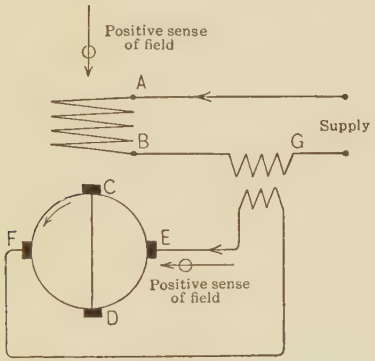


Fig. 15.

246. Figure 16 represents a Fynn polyphase synchronous induction motor (see A.I.E.E. *Journal*, August, 1924, pages 744–748). This motor under normal operating conditions has the characteristics of a polyphase synchronous motor. Below synchronous speed the performance of this motor is like that of a polyphase induction motor. The armature winding is series mesh-star connected. The commutator bars are connected to the mesh winding and the terminals of the star winding are connected to collector rings. Polyphase power is supplied to the rotor and not to the stator winding as in an ordinary polyphase motor. (*a*) Show by diagrams that, (1) the synchronizing torque

due to stator winding N is alternating, and (2) that the synchronizing torque due to stator winding M is pulsating and unidirectional. (*b*) Compare the relative magnitudes of the synchronizing torques produced by windings M and N.

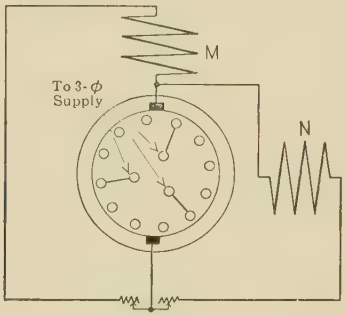


Fig. 16.

247. A certain phase-balancer (see Pender's "Handbook for Electrical Engineers," 1922 edition, pages 1110 to 1113) is used for balancing the output of a three-phase generator. The generator and the balancer are connected to three bus-bars numbered 1, 2, and 3. Three feeders, A, B, and C, supply the total load and are connected to the bus-bars 1, 2, and 3, respectively. The current in feeder A referred to the bus-bar voltage from bus-bar 1 to neutral is $104 + j75$ amperes; the current in feeder B referred to the same voltage is $-200 + j80$. (*a*) Calculate: (1) the current in feeder C; and (2) the value of the counter-phase rotational current required to balance the generator output. (*b*) Outline a method for calculating the rating of the synchronous motor and generator elements of the balancer.

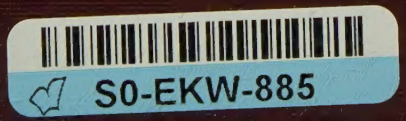
621.3133 B87

236

RIZONA

DO NOT REMOVE THIS
CARD FROM BOOK
POCKET. A FEE OF $1.00
WILL BE CHARGED FOR
LOSS OR MUTILATION

62179


S0-EKW-885